Mesons as Open Strings in a Holographic Dual of QCD

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based on: **arXiv:1005.0655**

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ハドロン スクエア @ 京大 7/3/2010

1 Introduction

Meson effective theory (traditional approach)

effective action consistent with chiral sym, hidden local sym.

$$
\begin{aligned}
S_{\text{4dim}} = \int d^4x \Bigg[& \left.\begin{aligned}
&\frac{f_\pi^2}{4}\,\mathrm{tr}[D_\mu U^\dagger D^\mu U] \\
&+L_1\left(\mathrm{tr}[D_\mu U^\dagger D U]\right)^2 + L_2\,\mathrm{tr}[D_\mu U^\dagger D_\nu U]\,\mathrm{tr}[D^\mu U^\dagger D^\nu U] \\
&+L_3\,\mathrm{tr}[D_\mu U^\dagger D^\mu U D^\nu U^\dagger D^\nu U] \\
&-iL_9\,\mathrm{tr}[F^L_{\mu\nu} D^\mu U^\dagger D^\nu U^\dagger + F^R_{\mu\nu} D^\mu U^\dagger D^\nu U] + L_{10}\,\mathrm{tr}[U^\dagger F^L_{\mu\nu} U F^{R\,\mu\nu}]
\end{aligned}\right\}\ \text{pion} \\
& +\frac{1}{2}\mathrm{tr}F^v_{\mu\nu}F^{v\,\mu\nu} + m_\rho^2\,\mathrm{tr}[(v_\mu - g^{-1}\beta_\mu)^2] \Bigg] \quad \longleftarrow \ \rho \text{ meson}
\end{aligned}
$$

$$
\begin{aligned}
-\frac{N_c}{24\pi^2}\int_{\text{4dim}} \Bigg[& \mathrm{tr}[(A_R dA_R + dA_R A_R + A_R^3)(U^{-1}A_L U + U^{-1}dU) - \mathrm{p.c.}] \\
& +\mathrm{tr}[dA_R dU^{-1} A_L U - \mathrm{p.c.}] + \mathrm{tr}[A_R(dU^{-1}U)^3 - \mathrm{p.c.}] \\
& +\frac{1}{2}\mathrm{tr}[(A_R dU^{-1}U)^2 - \mathrm{p.c.}] + \mathrm{tr}[U A_R U^{-1} A_L dU dU^{-1} - \mathrm{p.c.}] \\
& -\mathrm{tr}[A_R dU^{-1} U A_R U^{-1} A_L U - \mathrm{p.c.}] + \frac{1}{2}\mathrm{tr}[(A_R U^{-1} A_L U)^2] \\
& +C_1\,\mathrm{tr}[\alpha_L^3\alpha_R - \alpha_R^3\alpha_L] + C_2\,\mathrm{tr}[\alpha_L\alpha_R\alpha_L\alpha_R] \\
& +C_3\,\mathrm{tr}[F^v\alpha_L\alpha_R\alpha_L\alpha_R] \\
& +C_4\,\mathrm{tr}[F^L(\alpha_L\alpha_R - \alpha_R\alpha_L) - F^R(\alpha_R\alpha_L - \alpha_L\alpha_R)] \Bigg]
\end{aligned}
$$

$$
-\frac{N_c}{240\pi^2}\int_{\text{5dim}} \mathrm{Tr}(g d g^{-1})^5
$$

WZW term

$$+(\text{much more})$$

- A lot of terms
- A lot of parameters

$$
\begin{aligned}
&D_\mu U = \partial_\mu U - iA^L_\mu U + iUA^R_\mu \\
&U = \xi_L^\dagger \xi_R \\
&\beta_\mu = \frac{1}{2i}(\partial_\mu\xi_R\cdot\xi_R^\dagger + \partial_\mu\xi_L\cdot\xi_L^\dagger) \\
&D_\mu\xi_L = \partial_\mu\xi_L - igv_\mu\xi_L + i\xi_L A^L_\mu \\
&D_\mu\xi_R = \partial_\mu\xi_R - igv_\mu\xi_R + i\xi_R A^R_\mu \\
&\alpha_{L\mu} = \frac{1}{i}D_\mu\xi_L\cdot\xi_L^\dagger, \quad \alpha_{R\mu} = \frac{1}{i}D_\mu\xi_R\cdot\xi_R^\dagger
\end{aligned}
$$

"Top down approach" of holographic QCD

[Sakai-S.S. 2004]

1. Find a D-brane configuration that realizes QCD
2. Use the Gauge/String duality
3. Some approximation

Wait for the explanation

Meson effective theory

5 dim U(N_f) YM-CS theory in a curved space-time

$$S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$$k(z) = 1 + z^2$$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$$\kappa = \frac{\lambda N_c}{216\pi^3} \equiv a\lambda N_c \qquad h(z) = (1 + z^2)^{-1/3}$$

$(M_{\text{KK}} = 1 \text{ unit})$

- Just one line
- Just 2 parameters

M_{KK} ~ "cut off" scale

λ ~ bare coupling

5 dim YM-CS theory = 4 dim meson theory

$$A_\mu(x^\mu, z) = \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n\geq 0} \varphi^{(n)}(x^\mu)\phi_n(z)$$

complete sets

Chosen to diagonalize kinetic & mass terms of $B_\mu^{(n)}, \varphi^{(n)}$

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$$\boxed{S_{5\text{dim}}(A) = S_{4\text{dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)}$$

- reproduces old phenomenological models
  - Skyrme model [Skyrme 1961]
  - Vector meson dominance [Gell-Mann -Zachariasen 1961, Sakurai 1960]
  - Gell-Mann Sharp Wagner model [Gell-Mann -Sharp-Wagner 1962]
  - Hidden local symmetry [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]
- masses and couplings roughly agree with experiments.

Quantitative tests

meson mass

mass	ρ	a_1	ρ'
exp.(MeV)	776	1230	1465
our model	[776]	1189	1607

[T.Sakai-S.S. 04]

baryon static properties

baryon	our model	exp.
$\langle r^2\rangle^{1/2}_{I=0}$	0.742 fm	0.806 fm
$\langle r^2\rangle^{1/2}_{I=1}$	0.742 fm	0.939 fm
$\langle r^2\rangle^{1/2}_{A}$	0.537 fm	0.674 fm
$g_{I=0}$	1.68	1.76
$g_{I=1}$	7.03	9.41
g_A	0.734	1.27

[K.Hashimoto-T.Sakai-S.S. 08]

couplings in meson eff action

coupling	our model	experiment
f_π	[92.4 MeV]	92.4 MeV
L_1	0.584×10^{-3}	$(0.1\sim0.7)\times10^{-3}$
L_2	1.17×10^{-3}	$(1.1\sim1.7)\times10^{-3}$
L_3	-3.51×10^{-3}	$-(2.4\sim4.6)\times10^{-3}$
L_9	8.74×10^{-3}	$(6.2\sim7.6)\times10^{-3}$
L_{10}	-8.74×10^{-3}	$-(4.8\sim6.3)\times10^{-3}$
$g_{\rho\pi\pi}$	4.81	5.99
g_ρ	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	4.63 GeV	$2.8\sim4.2$ GeV

[T.Sakai-S.S. 05]

Today, I won't explain all these
See our papers

What about other mesons?

- mesons in PDG meson summary table ($N_f = 2$, Isovector)

parity

charge conjugation

spin

J^{PC}

mass (MeV)

$\triangle$ ... not established

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	985	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
$1^{+-}(b_1)$	1230					
$1^{-+}(\pi_1)$	1376	1653				
$2^{++}(a_2)$	1318	$1732^{\triangle}$				
$2^{-+}(\pi_2)$	1672	1895	$2090^{\triangle}$			
$3^{--}(\rho_3)$	1689	$1990^{\triangle}$	$2250^{\triangle}$			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	$2330^{\triangle}$					
$6^{++}(a_6)$	$2450^{\triangle}$					

obtained from 5 dim gauge field (massless mode of open string)

Q: Can we understand this table from string theory?

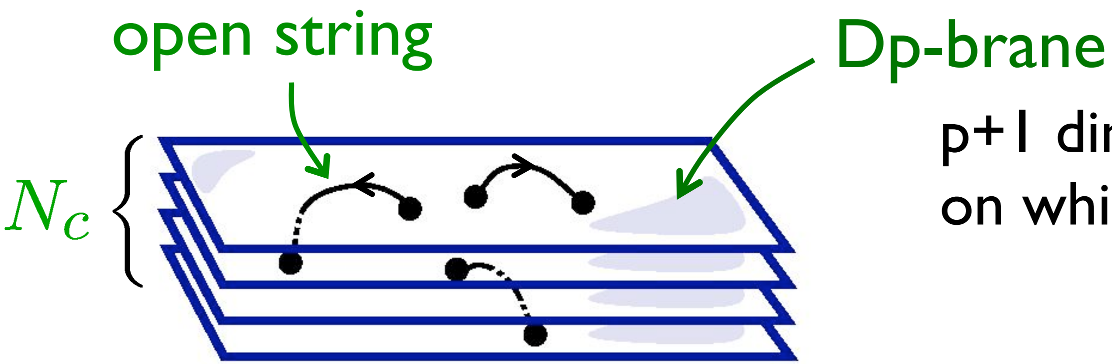
→ Consider massive modes (excited strings)

Plan

1. Introduction
2. Brief review of the model
3. Meson spectrum
4. Comparison with data
5. Discussion

2 Brief review of the model

● D-brane and Gauge theory



p+1 dimensional plane
on which open strings can end

a b → $(A_\mu)^{a}{}_{b}$ etc.

$a, b = 1 \sim N_c$

$U(N_c)$ gauge field

➡ (p+1) dim $U(N_c)$ gauge theory

Gauge / String duality

AdS/CFT

[Maldacena 97]

D3-brane

N=4 Super Yang-Mills

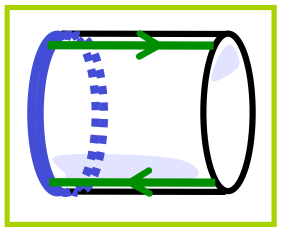
(gauge theory realized on the D3-brane)

dual

String theory in $AdS_5 \times S^5$

(SUGRA sol. corresponding to the D3-brane)

holographic QCD



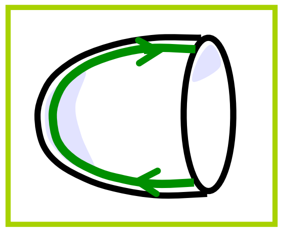
Some brane config.

1

2

QCD

dual



String theory in some curved b.g.

➡ “holographic QCD”

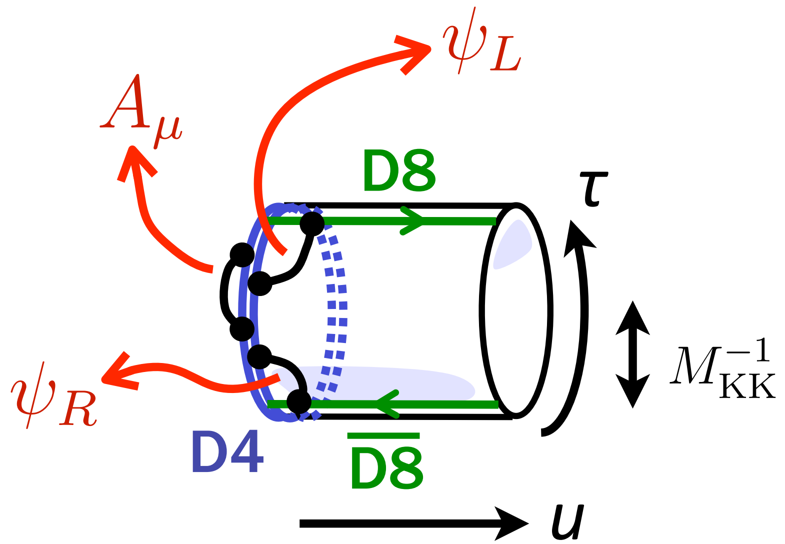
Brane configuration [T. Sakai and S.S. 04]

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 x N_C	○	○	○	○	○					
D8-$\overline{\text{D8}}$ x N_f	○	○	○	○		○	○	○	○	○

S^1 with ~~SUSY~~ b.c. [Witten 98]

$$\psi(x^\mu,\tau) = -\psi(x^\mu,\tau+2\pi M_{\rm KK}^{-1})$$

fermion



(radial direction of $x^{5\sim9}$)

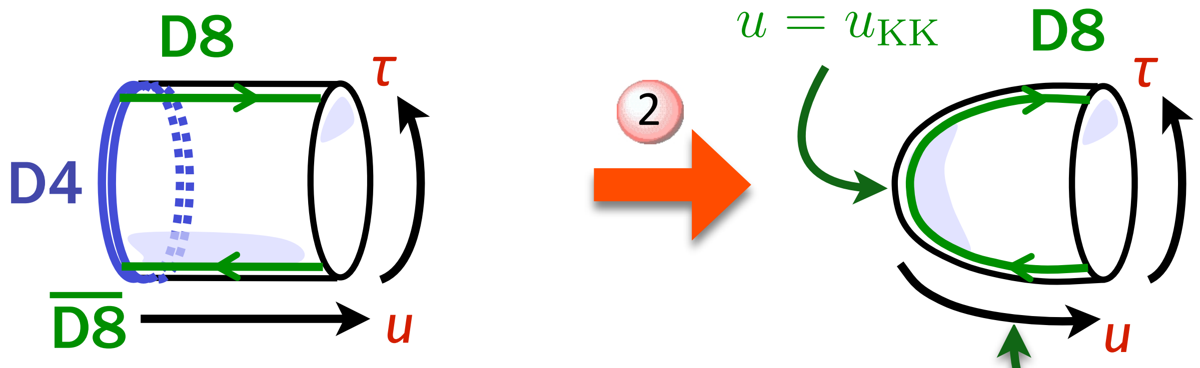
1

4 dim $U(N_C)$ QCD
with N_f massless quarks
(at low energy)

※ We will work in $M_{\rm KK}$=1 unit.

Holographic description

- replace D4 with the corresponding SUGRA solution
- D8 are treated as probe brane (assuming $N_c \gg N_f$)



$u \geq u_{\mathrm{KK}}$

$$\underset{x^{0\sim 3}}{\mathbf{R}^{1,3}} \times \underset{\tau}{S^1} \times \underset{(u,\,\theta^{1\sim 4})}{\mathbf{R}^5} \qquad S^4$$

$$\underset{x^{0\sim 3}}{\mathbf{R}^{1,3}} \times \underset{(u,\tau)}{\mathbf{R}^2} \times \underset{\theta^{1\sim 4}}{S^4} \qquad \text{(topology)}$$

metric : (Double Wick rotated black 4-brane solution)

$$ds^2 = H(u)^{-1/2}(-dt^2 + d\vec{x}^2 + f(u)d\tau^2) + H(u)^{1/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

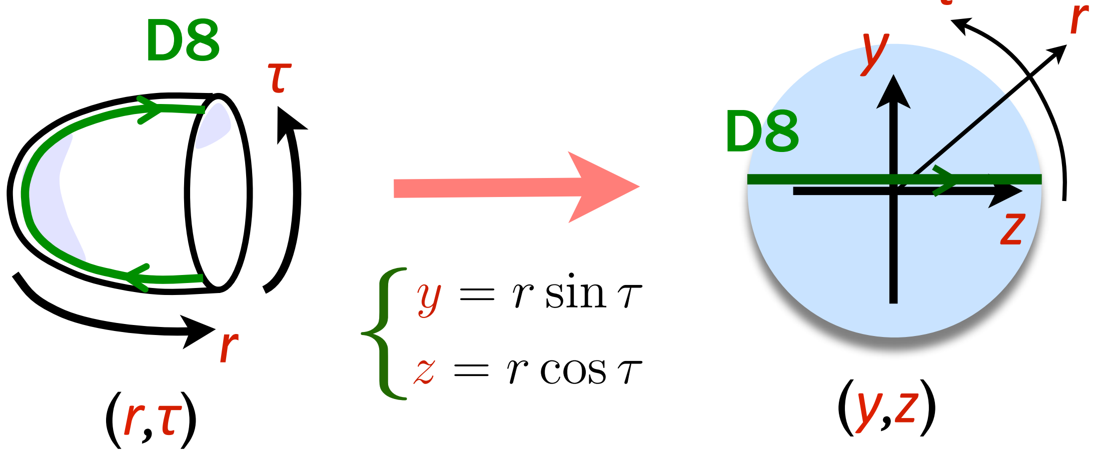
$$H(u) = \frac{R^3}{u^3} \qquad f(u) = 1 - \frac{u_{\mathrm{KK}}^3}{u^3} \qquad R^3 = \frac{\lambda l_s^2}{2} \qquad u_{\mathrm{KK}} = \frac{2}{9}\lambda l_s^2 \qquad (M_{\mathrm{KK}}\text{=1 unit})$$

$\lambda = g_{\mathrm{YM}}^2 N_c$: 't Hooft coupling

l_s : string length $(\to 0)$

- New coordinate:

$$u^3 = u_{\mathrm{KK}}^3 K(r) \qquad K(r) = 1 + r^2$$



$$\begin{cases} y = r \sin\tau \\ z = r \cos\tau \end{cases}$$

- Note:

$$\text{string length} \sim \lambda^{-1/2}$$

$$\text{string coupling} \sim \lambda^{3/2}/N_c$$

large λ ⇔ weakly curved background

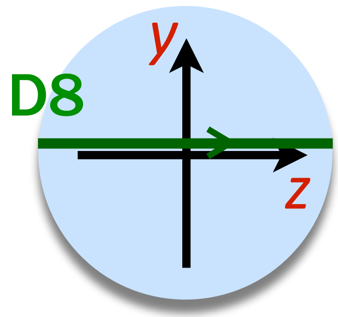
large N_c ⇔ weakly coupled string theory

Hadrons in the model

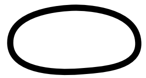
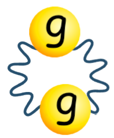
Topology of the space-time

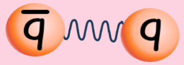
$$\underset{x^{0\sim3}}{\mathbf{R}^{1,3}} \times \underset{(z,y)}{\mathbf{R}^2} \times S^4$$

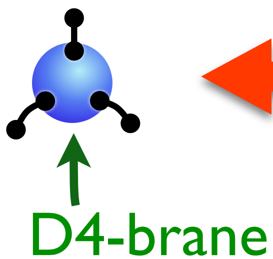
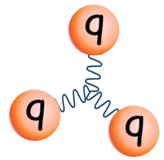
D8 are extended along $\underbrace{(x^\mu, z)}_{\text{5dim}} \times S^4$



particle-like objects:

- closed strings   glueballs

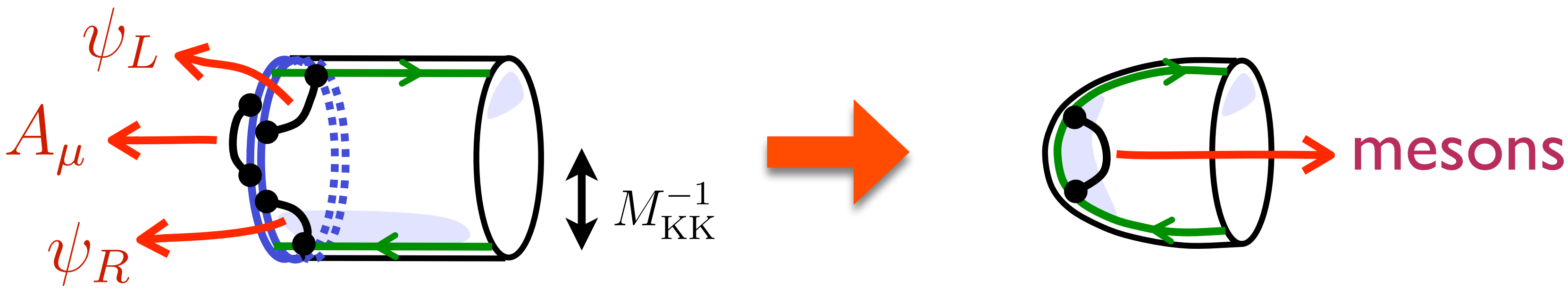
- open strings on D8   mesons

- D4 wrapped on S^4   baryons

today's topic

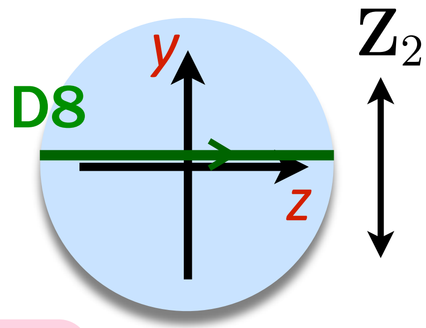
QCD mesons vs artifacts

- Our brane config. is invariant under $SO(5) \curvearrowright S^4$
- quarks and gluons are invariant under $SO(5)$
  (non-invariant states are massive modes)



- Bound states of quarks and gluons are $SO(5)$ invariant
  (non-invariant states are artifacts made by unwanted massive modes)

- Similarly, we can show that QCD mesons are
  invariant under Z_2 sym generated by $I_{y9}(-1)^{F_L}$
  I_{y9}: $(y, x^9) \rightarrow (-y, -x^9)$ $\quad$ $(\tau \rightarrow -\tau)$



→ **Consider $SO(5) \rtimes \mathbf{Z}_2$ invariant states**

3 Meson spectrum

Consider open strings attached on D8

Strategy

1. Consider flat space-time, (justified when $\lambda \gg 1$)
   and quantize the open strings attached on D8.

   space-time: $\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$ D8-brane: $(x^\mu, z) \times S^4$
   (topology) $x^{0\sim 3}$ (z, y) $x^{6\sim 9}$

   In the flat space-time limit,
   $S^4 \Rightarrow R^4$, $SO(5) \Rightarrow$ rotation and translation of $x^{6\sim 9}$

2. Pick up the $SO(5) \rtimes \mathbf{Z}_2$ invariant states.

   $\rightarrow$ reduced to 5 dim: (x^μ, z)

3. Recover the z dependence of the induced metric on D8.

- General rules for light-cone quantization (NS-sector)

  (light-cone direction $x^{\pm} = x^0 \pm x^1$)

  - Fock vacuum $\quad |0\rangle_{\mathrm{NS}}$

  - creation op. $\quad \psi^i_{-r}$ fermion $\quad \alpha^i_{-n}$ boson $\quad (i = 2, 3, \cdots, 9)$

    $(r = 1/2,\ 3/2, \cdots)$ $\quad (n = 1, 2, 3, \cdots)$

  - physical state $\quad \underbrace{\psi^{i_1}_{-r_1} \cdots \psi^{i_k}_{-r_k}}_{\text{odd}} \alpha^{j_1}_{-n_1} \cdots \alpha^{j_l}_{-n_l} |0\rangle_{\mathrm{NS}}$

  - mass $\quad m_0^2 = \dfrac{N}{\alpha'} \qquad N \equiv \sum_{s=1}^{k} r_s + \sum_{t=1}^{l} n_t - \dfrac{1}{2}$

※ We will not consider R-sector,
since there is no $SO(5)$ invariant states in R-sector.

- **Massless mode** (N=0)
  - $\psi^I_{-1/2}|0\rangle_{\rm NS} \quad (I=2,3,z)$ → 5 dim gauge field $A_\mu,\ A_z$
  - $\psi^A_{-1/2}|0\rangle_{\rm NS} \quad (A=y,6,7,8,9)$ → not invariant under $SO(5)\rtimes \mathbf{Z}_2$

- **KK decomposition along z direction**

Recovering the curved background, we obtain
5 dim $U(N_f)$ YM-CS theory in a curved space-time.

$$S_{5\rm dim}=\kappa\int d^4x dz\,{\rm Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2+K(z)F_{\mu z}^2\right)+\frac{N_c}{24\pi^2}\int_5\omega_5(A) \qquad K(z)=1+z^2$$

$$A_\mu(x^\mu,z)=\sum_{n=1}^{\infty}B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$A_z(x^\mu,z)=\sum_{n=0}^{\infty}\varphi^{(n)}(x^\mu)\phi_n(z)$$

complete sets

	$B_\mu^{(1)}$	$B_\mu^{(2)}$	$B_\mu^{(3)}$	…	$\varphi^{(0)}$	$\varphi^{(1)}$	…
J^{PC}	1^{--}	1^{++}	1^{--}	…	0^{-+}	eaten	
	ρ	a_1	ρ'	…	π		

[T.Sakai and S.S. 04]

$$-K(z)^{1/3}\partial_z(K(z)\partial_z\psi_n(z))=m_n^2\psi_n(z) \qquad \phi_n(z)=\partial_z\psi_n(z)$$

$\psi_n(z)$: eigenfunction $\quad m_n^2$: eigenvalue ⇒ mass^2 of $B_\mu^{(n)}$

First excited massive modes (N=1)

$SO(5) \rtimes \mathbf{Z}_2$ invariant states:

$(I = 2, 3, z)$

- $\psi^{I}_{-3/2}|0\rangle_{\rm NS}$
- $\alpha^{(I}_{-1}\psi^{J)}_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^{[I}_{-1}\psi^{J]}_{-1/2}|0\rangle_{\rm NS}$
- $\psi^{I}_{-1/2}\psi^{J}_{-1/2}\psi^{K}_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^{y}_{-1}\psi^{y}_{-1/2}|0\rangle_{\rm NS}$
- $\underbrace{\sum_{a=6,7,8,9}}_{S^4} \alpha^{a}_{-1}\psi^{a}_{-1/2}|0\rangle_{\rm NS}$

5dim field	$SO(4)$ little gr
h_{MN} $(M, N = 1, 2, 3, z)$	$\square\square$
A_{MNP}	$\begin{array}{c}\square\\\square\\\square\end{array}$
$\varphi^{[1]}$	1
$\varphi^{[2]}$	1

KK decomposition along z direction

$$h_{MN}(x^\mu, z) = \sum_{n=0}^{\infty} h_{MN}^{(n)}(x^\mu)\phi_n(z) \qquad \text{etc.}$$

lowest modes (n=0):

	$h_{ij}^{(0)}$	$h_{iz}^{(0)}$	$h_{zz}^{(0)}$	$A_{ijk}^{(0)}$	$A_{ijz}^{(0)}$	$\varphi^{[1,2](0)}$
J^{PC}	2^{++}	1^{+-}	0^{++}	0^{-+}	1^{--}	0^{++} x 2

(i , j , k = 1, 2, 3)

Second excited massive mode (N=2)

lowest modes (n=0):

J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

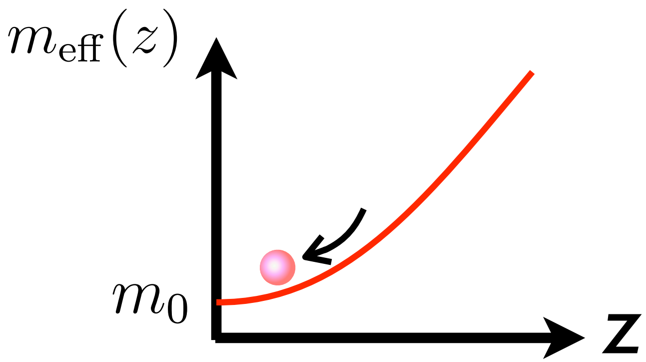
Mass formula for $N>0$ states (naive shortcut)

- Flat space-time limit: $\boxed{m_0^2 = \dfrac{N}{\alpha'}}$ $\left(\alpha'^{-1} = \dfrac{4}{27}\lambda M_{\rm KK}^2\right)$

  (mass[2] for 5 dim field)

  $N = 0, 1, 2, \cdots$: excitation level

- recovering the z dependence of the metric,



$$S = -m_0 \int \sqrt{g_{tt}}\, dt = \underbrace{-m_0 (1+z^2)^{1/4}}_{m_{\rm eff}(z)} \int dt$$

(mass for 4 dim mesons)

$$\boxed{M_n \simeq \underbrace{m_0}_{\mathcal{O}(\lambda^{1/2})} + \underbrace{\frac{1}{\sqrt{2}}\left(n+\frac{1}{2}\right)}_{\text{harmonic oscillator approx.}} M_{\rm KK} + \mathcal{O}(\lambda^{-1/2})} \qquad n = 0, 1, 2, \cdots$$

- More careful analysis shows that the $O(1)$ term is not affected by the RR-flux, α' correction, etc.

4 Comparison with data

Now we are ready to compare our results with the experimental data

But, don't trust too much !

- $1/N_c,\ 1/\lambda$ corrections may be large.
- We know α' does not agree well with lattice and experiment, if we use m_ρ and f_π as inputs.
- quarks are massless in our model.
- The model deviates from real QCD at high energy $\sim M_{KK} \sim 1$ GeV

But, don't be too pessimistic.

- The effect of "cut off" at M_{KK} is much milder than lattice cut off.
- Remember "quench approximation" works in lattice QCD
- At least, we should not give up before trying.

Massless mode

[T.Sakai and S.S. 04]

	$B_\mu^{(1)}$	$B_\mu^{(2)}$	$B_\mu^{(3)}$	$B_\mu^{(3)}$	...	$\varphi^{(0)}$
J^{PC}	1^{--}	1^{++}	1^{--}	1^{++}	...	0^{-+}
	ρ	a_1	ρ'	a'_1	...	π
mass (MeV)	[776]	1189	1607	2024	...	0

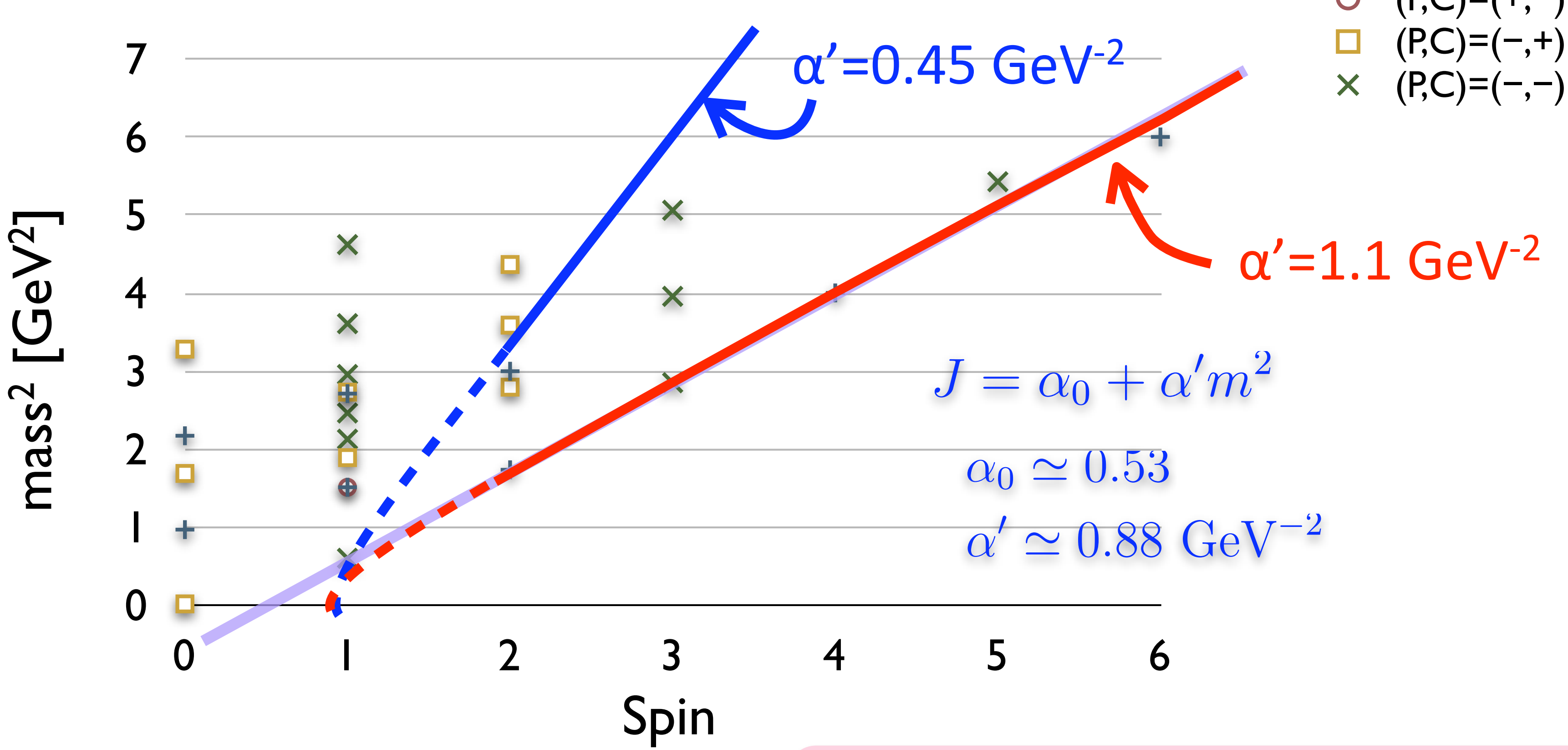
used to fix
M_{KK}=949 MeV

?

experiment:

$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				

Regge trajectory



$$M_n \simeq \sqrt{\frac{N}{\alpha'} + \frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right)} M_{\mathrm{KK}} \quad \Rightarrow \quad J \simeq 1 + \alpha' M^2 - \frac{\alpha'}{\sqrt{2}} M_{\mathrm{KK}} M + \mathcal{O}(\lambda^{-1})$$

(for $N \geq 1$)

$N = J - 1, \;\; n = 0$

- If we use f_π to fit λ, we obtain α' = 0.45 GeV^{-2}. This is unfortunately too small.
- If we set α'=1.1 GeV^{-2} we get very good fit.

First excited states (*N*=1, *n*=0)

J^{PC}	2^{++}	1^{+-}	1^{--}	0^{-+}	0^{++} x 3

2^{++}, 1^{+-}, 0^{-+}, 0^{++} cannot be *N*=0

$\Rightarrow$ good candidates for *N*=1

?

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	*~~985~~	1474				
$1^{--}(\rho)$	776	1459	1570△	1720	1900△	2150△
$1^{++}(a_1)$	1230	1647△				
$1^{+-}(b_1)$	1230					
$1^{-+}(\pi_1)$	1376	1653				
$2^{++}(a_2)$	1318	1732△				
$2^{-+}(\pi_2)$	1672	1895	2090△			
$3^{--}(\rho_3)$	1689	1990△	2250△			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	2330△					
$6^{++}(a_6)$	2450△					

: *N*=0

: *N*=1

- degenerate around 1300 MeV
- *a_0(980) is considered to be a four quark state.

Second excited states (N=2, n=0)

J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	~~985~~	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
$1^{+-}(b_1)$	1230					
$1^{-+}(\pi_1)$	~~1376~~ *	1653				
$2^{++}(a_2)$	1318	$1732^{\triangle}$				
$2^{-+}(\pi_2)$	1672	1895	$2090^{\triangle}$			
$3^{--}(\rho_3)$	1689	$1990^{\triangle}$	$2250^{\triangle}$			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	$2330^{\triangle}$					
$6^{++}(a_6)$	$2450^{\triangle}$					

: N=0

: N=1

: N=2

- ★: prediction ?
- degenerate around 1700 MeV
- * π_1 (1400) is claimed to be a four quark state. (could be hybrid)

Summary

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	~~985~~	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
$1^{+-}(b_1)$	1230					
$1^{-+}(\pi_1)$	~~1376~~	1653				
$2^{++}(a_2)$	1318	$1732^{\triangle}$				
$2^{-+}(\pi_2)$	1672	1895	$2090^{\triangle}$			
$3^{--}(\rho_3)$	1689	$1990^{\triangle}$	$2250^{\triangle}$			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	$2330^{\triangle}$					
$6^{++}(a_6)$	$2450^{\triangle}$					

: N=0

: N=1

: N=2

: 4 quarks

I think this is non-trivial.

What do you think?

5 Discussion

- Mesons are Strings
- Wikipedia says:

> **Problems and controversy**
>
> Although string theory comes from physics, some say that string theory's current untestable status means that it should be classified as more of a mathematical framework for building models as opposed to a physical theory.
>
> ..... Yet, for all this activity, not a single new testable prediction has been made, not a single theoretical puzzle has been solved. ....

Don’t criticize string theory in this way anymore !