（3） ニュートリノ質量と
宇宙の構造形成、宇宙背景放射ゆらぎ

Taka KAJINO

National Astronomical Observatory

Dept of Astronomy, Grad School of Science, University of Tokyo

kajino@nao.ac.jp, http://www.cfca.nao.ac.jp/~kajino/

Neutrino Physics in Cosmology

● Neutrino Mass

(Lesgourgues and Pastor 2006)

ν-less double β-decay : $|\sum U^2_{e\beta} m_\beta| < 1\sim6$ eV ⟶ 0.1~0.05 eV !? (future)

CMB and LSS constraint from cosmological parameter-fit (at least 8) :

$\Sigma m_\nu < 1.3$ eV (2σ C.L.) ⟹ $\Omega_\nu h^2 < 0.013$

WMAP-5yr (Komatsu et al. 2008)

● Neutrino Anisotropic Stress

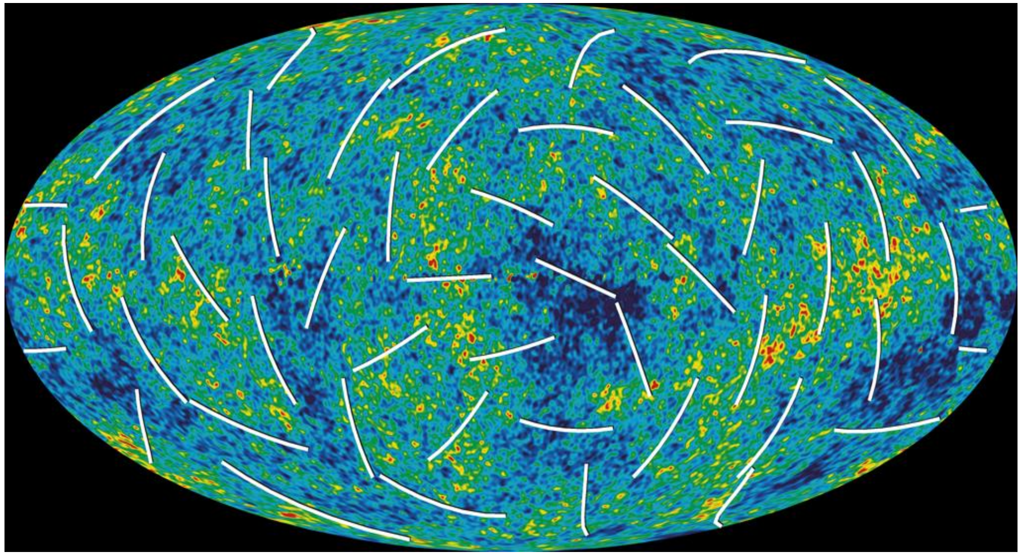
CMB is strongly affected by:

— integrated Sachs-Wolfe

— neutrino free streaming

but even generated by :

— compensation mode of neutrino anisotropic stress (π_ν) and another primordial source of extra anisotropic stress (π_{ext}) .

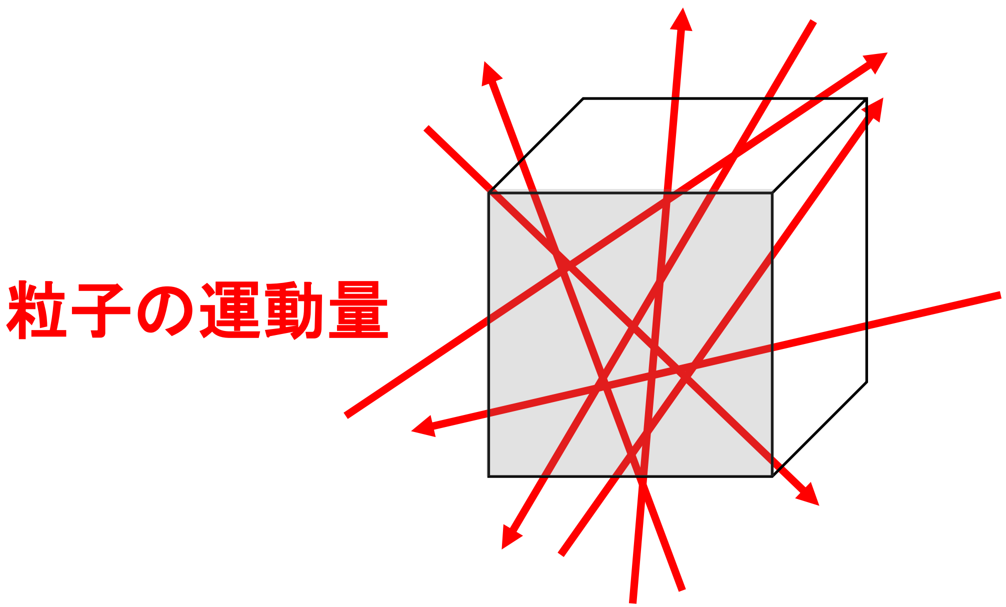


http://lambda/gsfc.nasa.gov/

非等方ストレス(π)とは？

ー運動量の非等方な流れ。

ー非相対論的物質(baryon, CDM)では無視できるが、
相対論的物質(photon, neutrino)では無視できない。



ーエネルギー運動量テンソルのtraceless成分。

エネルギー運動量テンソル

$$\delta G^{\mu\nu} = 8\pi G \delta T^{\mu\nu}$$

エネルギー密度

運動量密度

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

空間成分(ストレス成分) ： j-軸に垂直な面を通る運動量のi-成分

☆ Trace part ＝ 圧力

☆ Traceless part ＝ 非等方ストレス

Anisotropic Stress：磁場の例

Energy density

$$T_{\mu\nu} = \begin{pmatrix} \frac{B^2}{2} & 0 & 0 & 0 \\ 0 & -B_xB_x + \frac{B^2}{2} & -B_yB_x & -B_zB_x \\ 0 & -B_yB_x & -B_yB_y + \frac{B^2}{2} & -B_zB_y \\ 0 & -B_zB_x & -B_zB_y & -B_zB_z + \frac{B^2}{2} \end{pmatrix} \propto a^{-4}$$

Stress part (space-space part)

$\mathbf{B} \propto a^{-2}$

Trace part $\quad \frac{1}{3}\delta_{ij}\mathrm{Tr}(T_{ij})$ **Pressure**

Traceless part

$$\rho_\gamma \pi_B^{(0)} = -\frac{3}{2}(\hat{k}_i\hat{k}^j - \frac{1}{3}\delta_i{}^j)T_B{}^i{}_j(k)$$

Anisotropic Stress

Extra Anisotropic Stress π_{ext}（γ,ν以外の候補）

☆初期宇宙磁場（乱流起源） → 銀河･銀河団中の磁場の起源！？

磁場の強度

銀河団磁場 ･･･ 10^{-7}~10^{-6}G

銀河団の中の密度/宇宙の平均密度･･･ 10^{2}~10^{3}

$\therefore\ B_{prim} \sim B_{cl}(\rho/\rho_{cl})^{2/3} \sim 10^{-9}$G

☆Dark Radiation（in Brane World Cosmology）

我々の宇宙は高次元中のbrane？

→ Einstein方程式に修正項(dark radiation)

String-theory motivated Ekpyrotic/Cyclic model ➡ CMB Power Spectrum becomes too BLUE !

(Lehners & Steinhardt 2008; Khoury et al. 2001; Steinhardt & Turok 2002; Lehners et al. 2007)

Without consideration before ν-decoupling！

これまでの研究で、neutrino decoupling 以前のExtra Anisoropic Stress による曲率揺らぎ(scalar mode)の成長は考えられていなかった。（tensor mode は先行研究あり。）

目的

Critical Role of Neutrino Anisotropic Stress in CMB

⬇

インフレーションの仮定＝“ν脱結合期以前の曲率揺らぎを a priori に与える”ことがCMBにもたらす効果を解明する。

宇宙の揺らぎの進化

ー 宇宙はほぼ一様等方・平坦

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

ー 宇宙初期に僅かに曲率揺らぎが存在

→ 揺らぎが成長して宇宙の構造の種になる。

ー 光の速度は有限、情報の伝達距離に限界:horizon

$$\tau = \int_0^t \frac{dt'}{a(t')}$$

"conformal time" 時間を表す。

揺らぎのスケールがhorizonの中に入る(kτ>1)と、物理的に相互作用する

→ 揺らぎが成長

→ 揺らぎの密度、運動量、非等方ストレスなどを計算

→ CMBの理論値

一次摂動方程式

摂動 Einstein 方程式と Boltzmann 方程式を解く。

$$\tau^2 H_T^{(m)\prime\prime} + 2\tau H_T^{(m)\prime} = 3(R_\nu \pi_\nu^{(m)} + R_\gamma \pi_{\rm ex}^{(m)})$$
$$\pi_\nu^{(m)\prime} = -8H_T^{(m)\prime}/15,$$

ホライズンの外 (kτ<<1) に注目。 $H_T^{(0)} = \eta$

$$\tau^3 \frac{d^3\eta}{d\tau^3} + 6\tau^2 \frac{d^2\eta}{d\tau^2} + \left(6 + \frac{1}{3}k^2\tau^2\right)\tau\frac{d\eta}{d\tau} + \frac{2}{3}k^2\tau^2\eta$$
$$= -2(R_\nu\pi_\nu + R_\gamma\pi_{\rm ex}) - \tau R_\nu \frac{d\pi_\nu}{d\tau}$$

$$\frac{d^2\pi_\nu}{d(\ln\tau)^2} + \frac{d\pi_\nu}{d\ln\tau} + \frac{8}{5}R_\nu\pi_\nu = -\frac{8}{5}R_\gamma\pi_{\rm ex}$$

$$\left(\begin{aligned} R_\nu &= \frac{\rho_\nu}{\rho_\nu + \rho_\gamma} \\ R_\gamma &= \frac{\rho_\gamma}{\rho_\nu + \rho_\gamma} \end{aligned}\right)$$

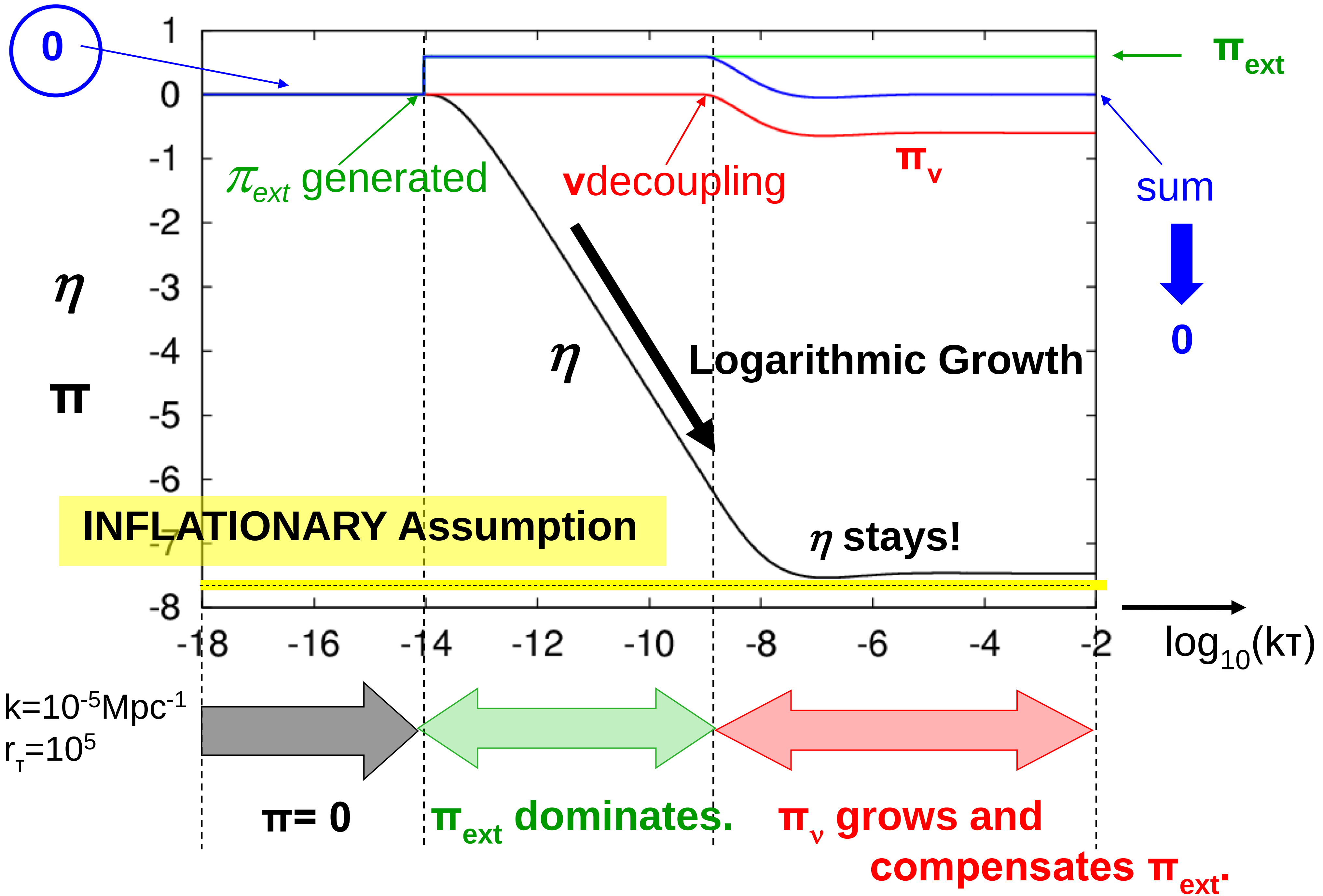
非等方ストレス(π)の関係する物理過程

物理過程	Standard Model (Inflation)	Extra Anisotropic Stress Model
Inflation? (~10^{15}GeV?) ↓ 曲率揺らぎの生成 $\nu_e + \bar{\nu}_e \leftrightarrow e^- + e^+$ など	**散乱による等方化を仮定 π=0**	**非等方ストレスが存在する。**
neutrino 脱結合(~1MeV, 1s) ↓ neutrinoが自由粒子 トムソン散乱	**π_νが成長**	**π_νが成長**
recombination (z~1000) ↓ photonも自由粒子	**π_γも成長**	**π_γも成長**

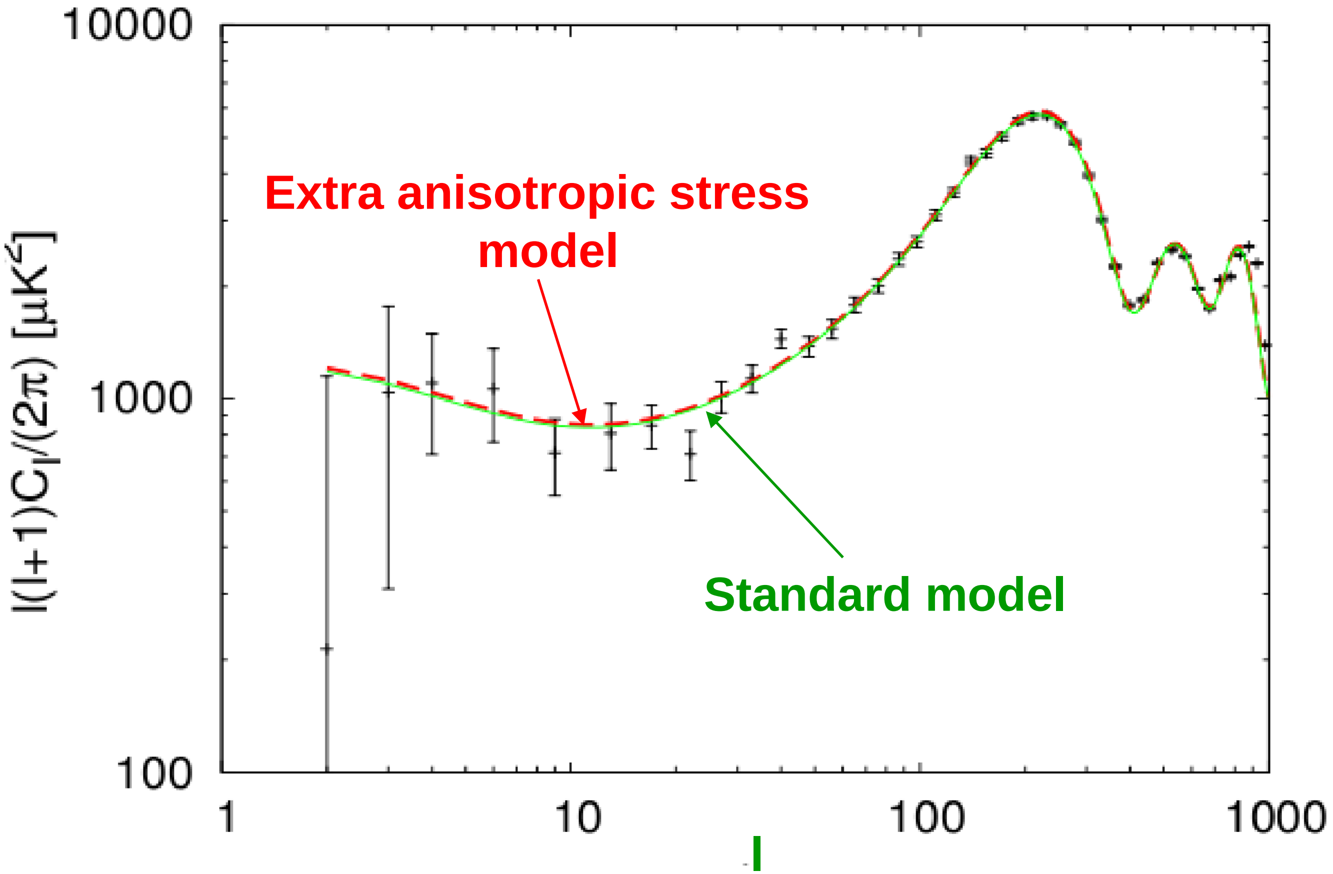
☆これまで neutrino decoupling 以前のExtra Anisotropic Stressによる曲率揺らぎ(scalar mode)の成長は考えられていなかった。

Evolution of η and π_i

Kojima, Kajino and Mathews, JCAP (2010)



CMB from Neutrino & Extra Anisotropic Stress



Kojima, Kajino and Mathews, JCAP (2010)

$|\pi_{ex}| \sim 8.4 \times 10^{-6}$

$r_\tau = 10^{18}$

Spectral index is set equal to be WMAP-best fit value.

Extra Anisotropic Stress Model is NOT an alternative to INFLATION !

— Curvature perturbation is generated by Extra Anisotropic Stress without assuming inflation-driven (pre-Big-Bang) perturbation.

— Future observation of non-Gaussianity is desirable to constrain the nature of Extra Anisotropic Stress.

— Primordial magnetic field could not be the full source of Extra Anisotropic Stress because of the constrained small Gaussianity.

Lewis 2004; Mack 2002; Challinor 2004; Kahniashvili & B. Ratra 2005; Kosowskyet al. 2005; Yamazaki et al. 2005 – 2009; Kojima et al. 2008 – 2009.

Observations of Magnetic Fields on Large Scales

☆ There are magnetic fields in clusters of galaxies.
☆ The amplitude is estimated to be ~1.0 μG.

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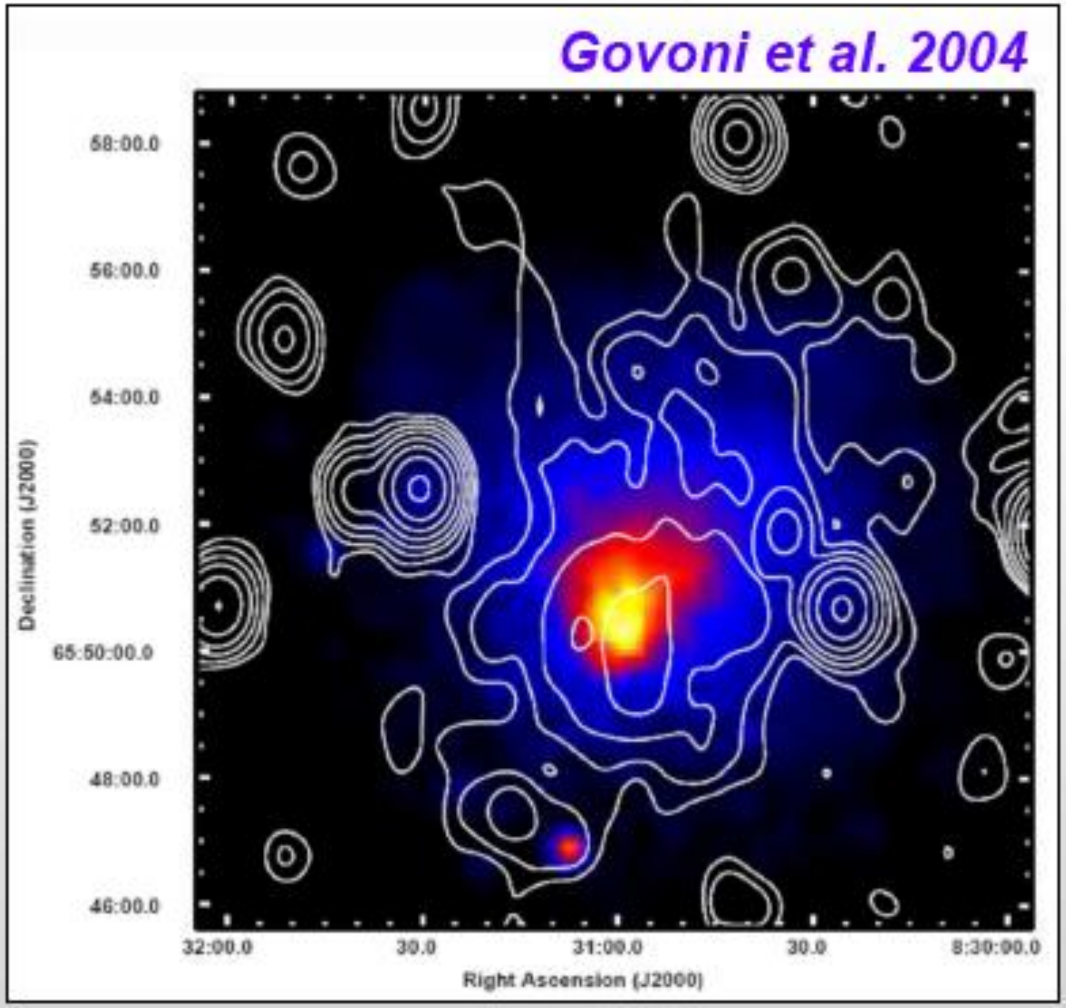
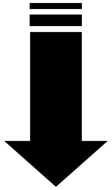


(Moffet & Birkinshaw 1989, Jones et al. 1996, Giovannini & Feretti 2000, Feretti et al. 2004)

Galaxy Clusters:

B ~ 1.0 μG



B ~ 1.0 nG
(Initial Seed)

How could primordial magnetic field (PMF) have been generated?

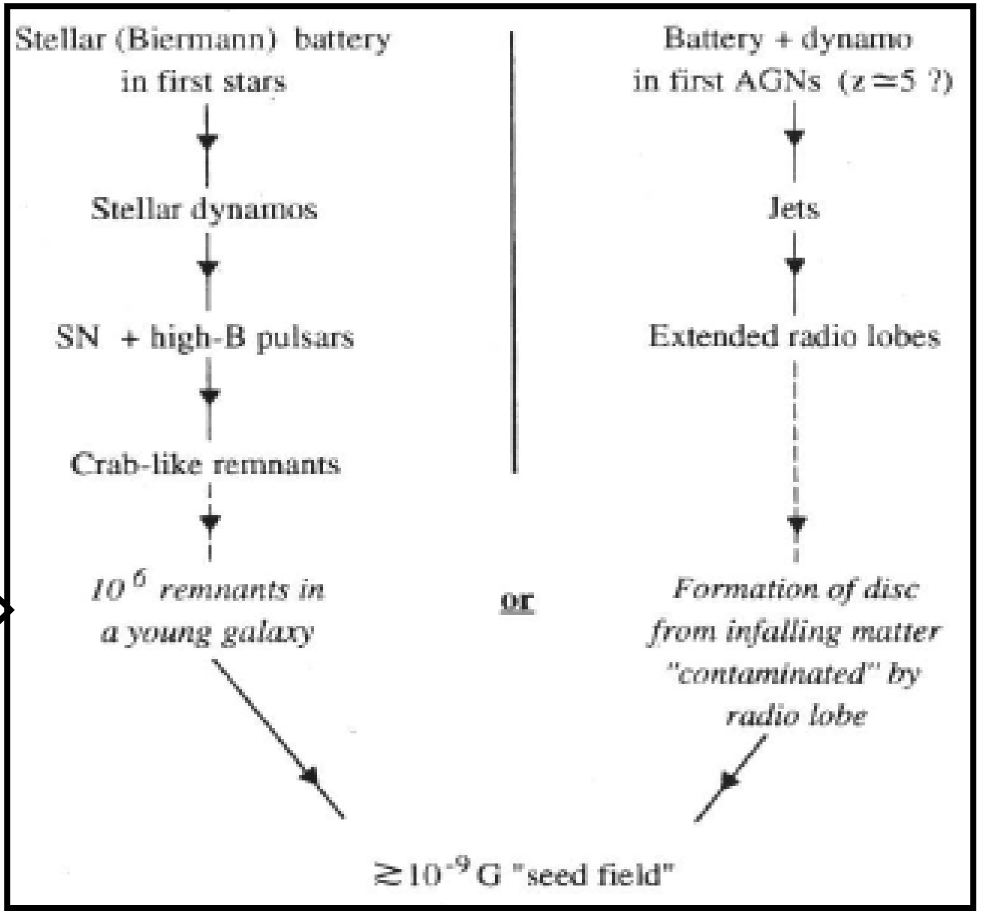
● Cosmological

- **Inflation** (Turner & Widraw 1988; Ratra 1992)
- **EW Baryogenesis via Sphaleron Transitions**
  $\theta_W \neq 0$, then $\mu_M \neq 0$
  (Nambu 1977,
  Hindmarsh & James 1994)
- **Dynamics of Recombination**
  (Ichiki et al. 2005)

vs.

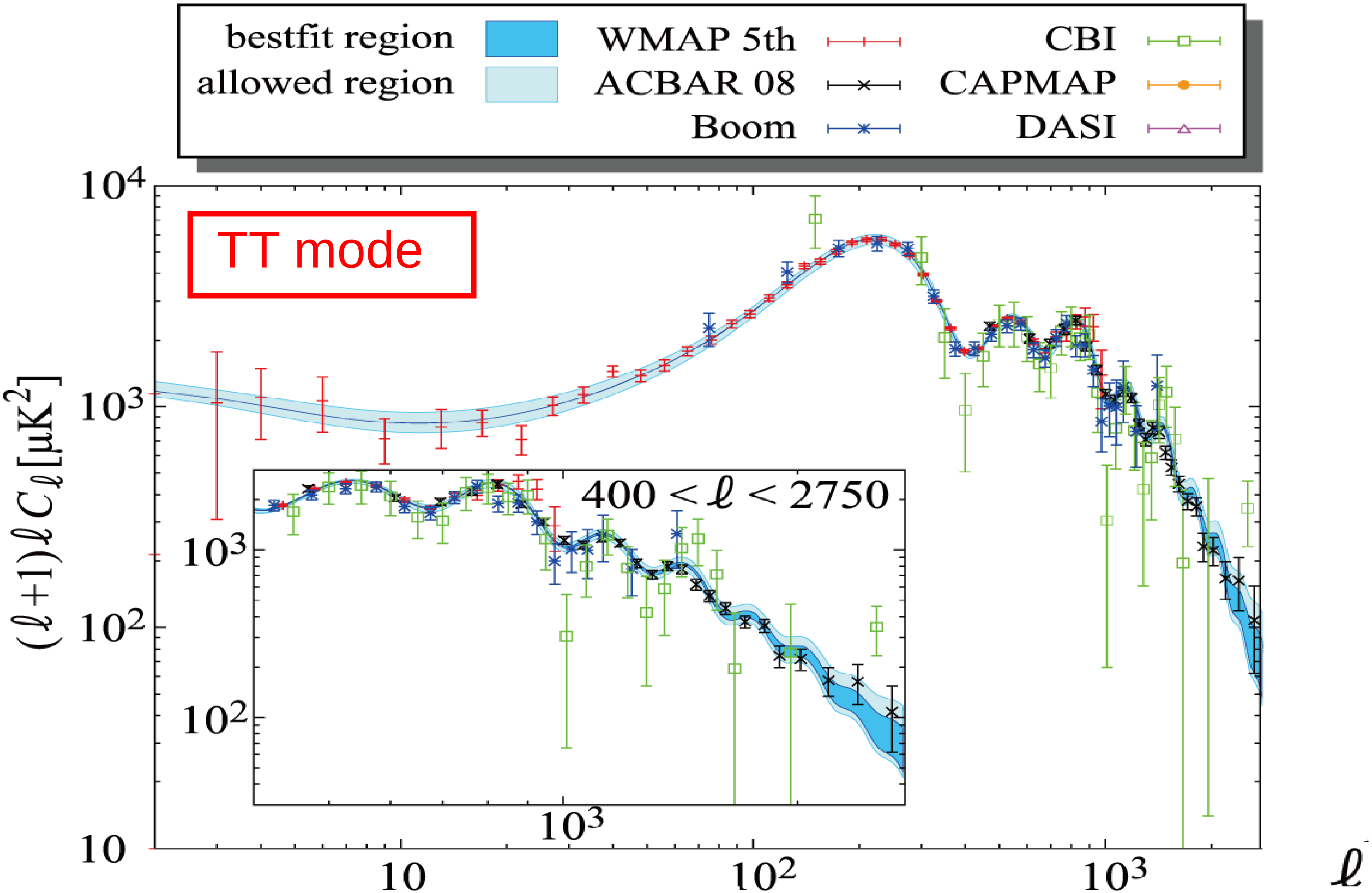
● Galactic or Stellar Sources + Dynamo in Post-Recombination

(M. Rees 1994)



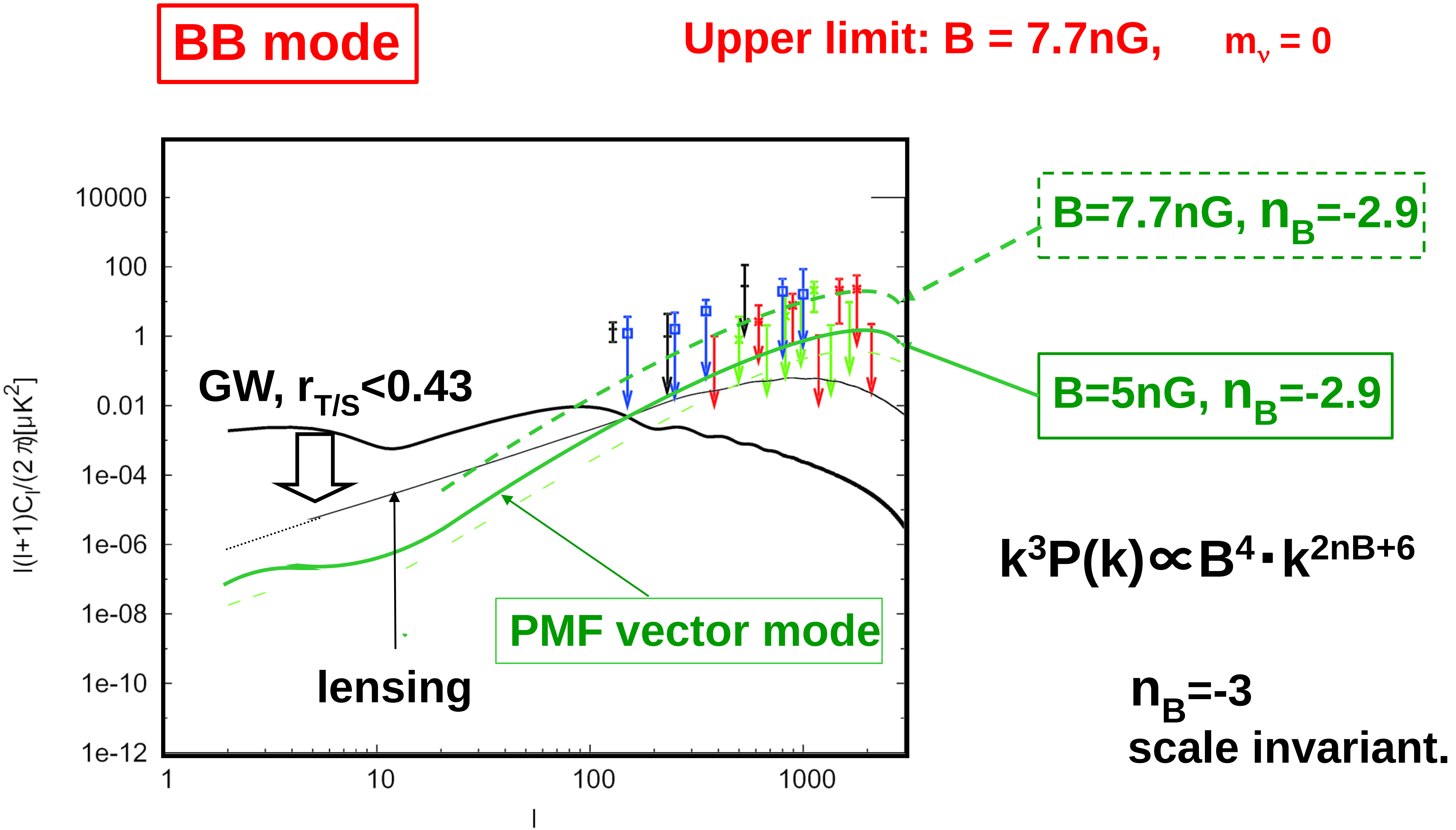
CMB Power Spectrum & Polarization with PMF

Yamazaki, Ichiki, Kajino, Mathews PR D77, 043005 (2008); PRD (2010) in press.



Expected Presence of Primordial Magnetic Field (PMF)

Yamazaki, Ichiki & Kajino, ApJ 825 (2006), L1.



$k^3P(k) \propto B^4 \cdot k^{2nB+6}$

n_B=-3
scale invariant.

Constraint on Cosmological Parameters

MCMC likelihood analysis

$\Omega_b h^2$

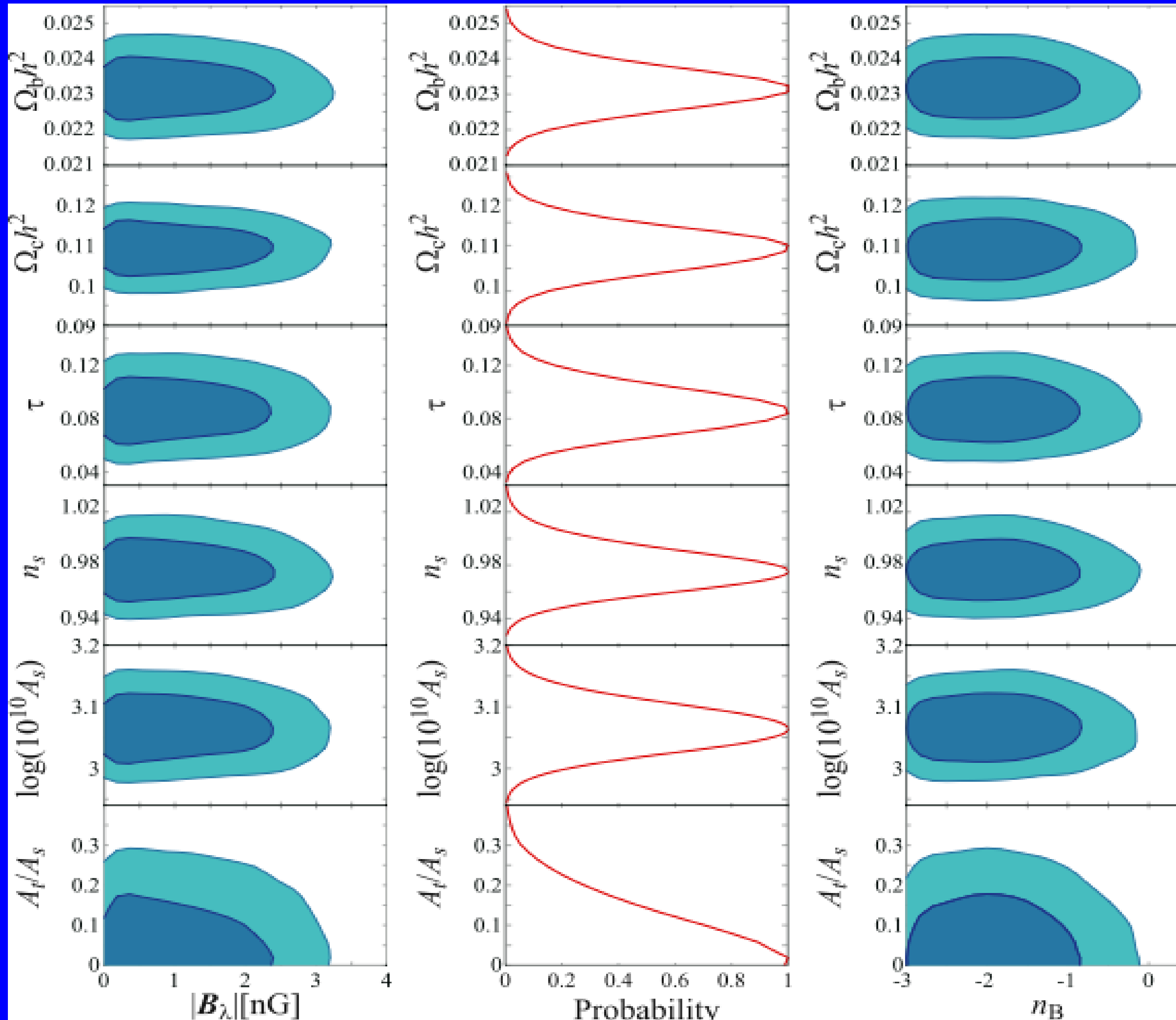
$\Omega_c h^2$

τ

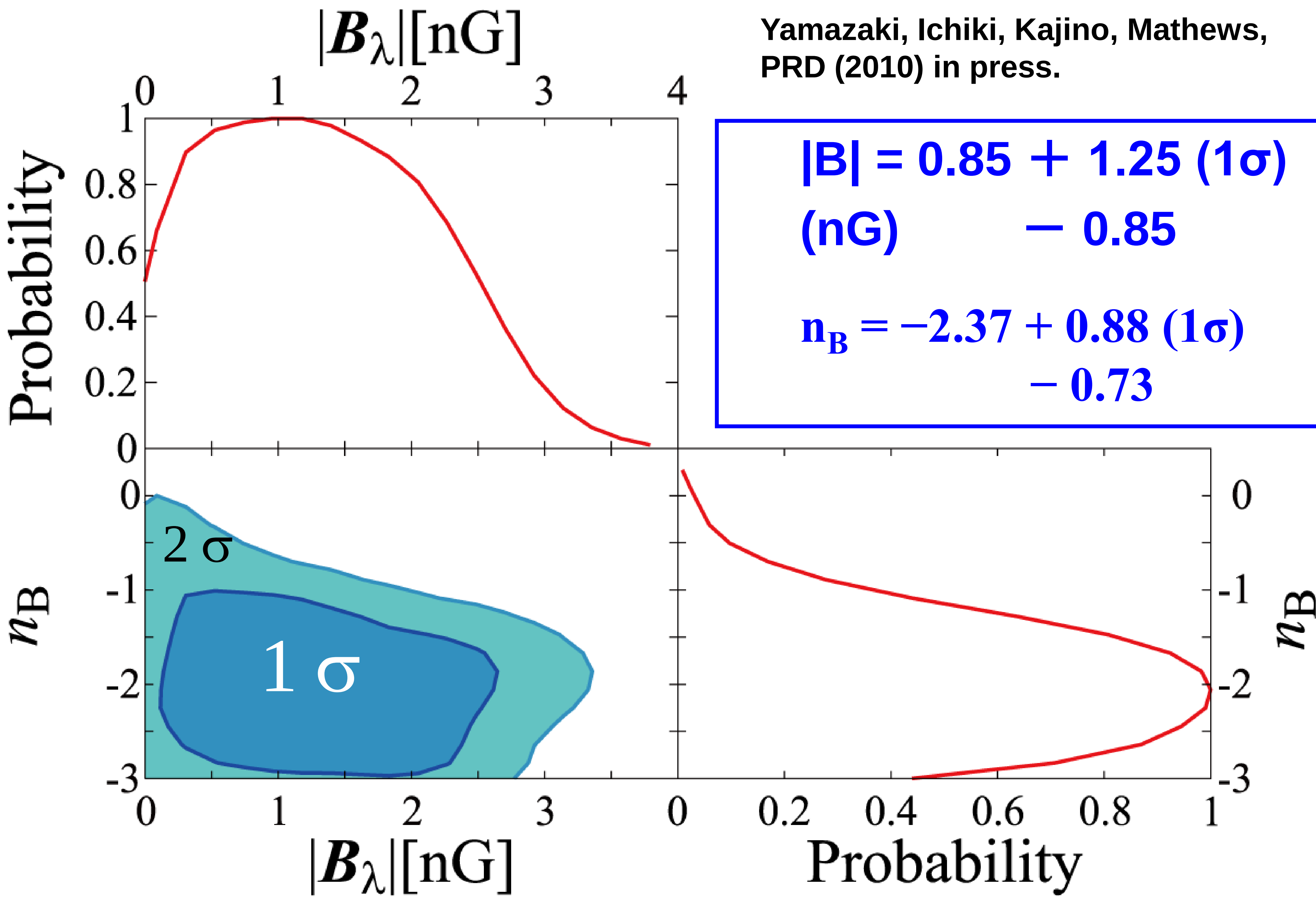
n_s

$\log(A_s)$

A_t/A_s



Constraint on parameters of PMF



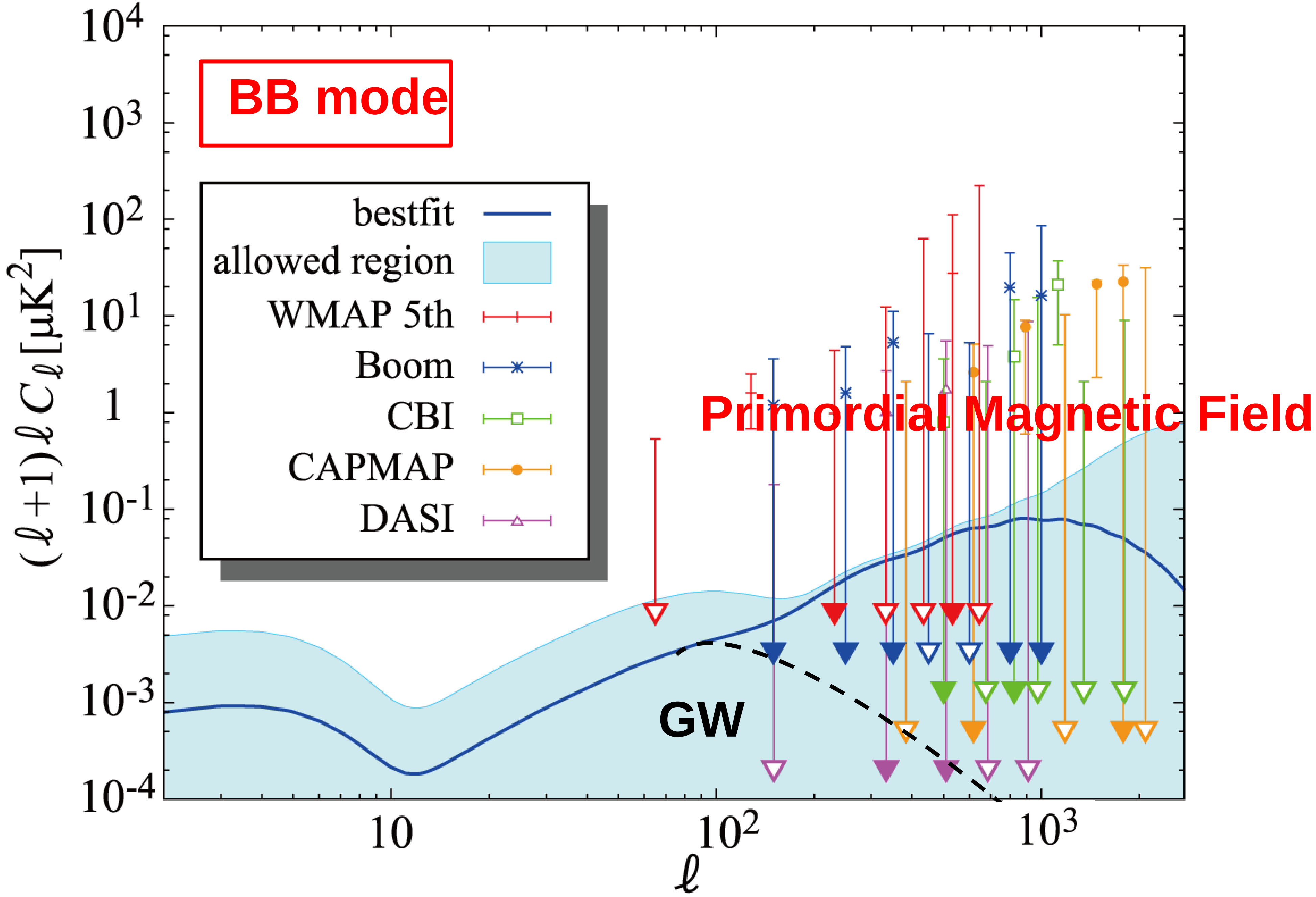
Yamazaki, Ichiki, Kajino, Mathews, PRD (2010) in press.

$|B|$ (nG) $= 0.85 \, {}^{+\,1.25}_{-\,0.85}$ (1σ)

$n_B = -2.37 \, {}^{+\,0.88}_{-\,0.73}$ (1σ)

Prediction of CMB-Polarization : BB mode

Yamazaki, Ichiki, Kajino, Mathews, PRD (2010) in press.



Neutrino Mass Effects

CMB anisotropies and polarization are influenced by

- Integrated Sachs-Wolfe Effect
- Free Streaming Effect
- Compensation effect of anisotropic stress of magnetic field & neutrinos

↓

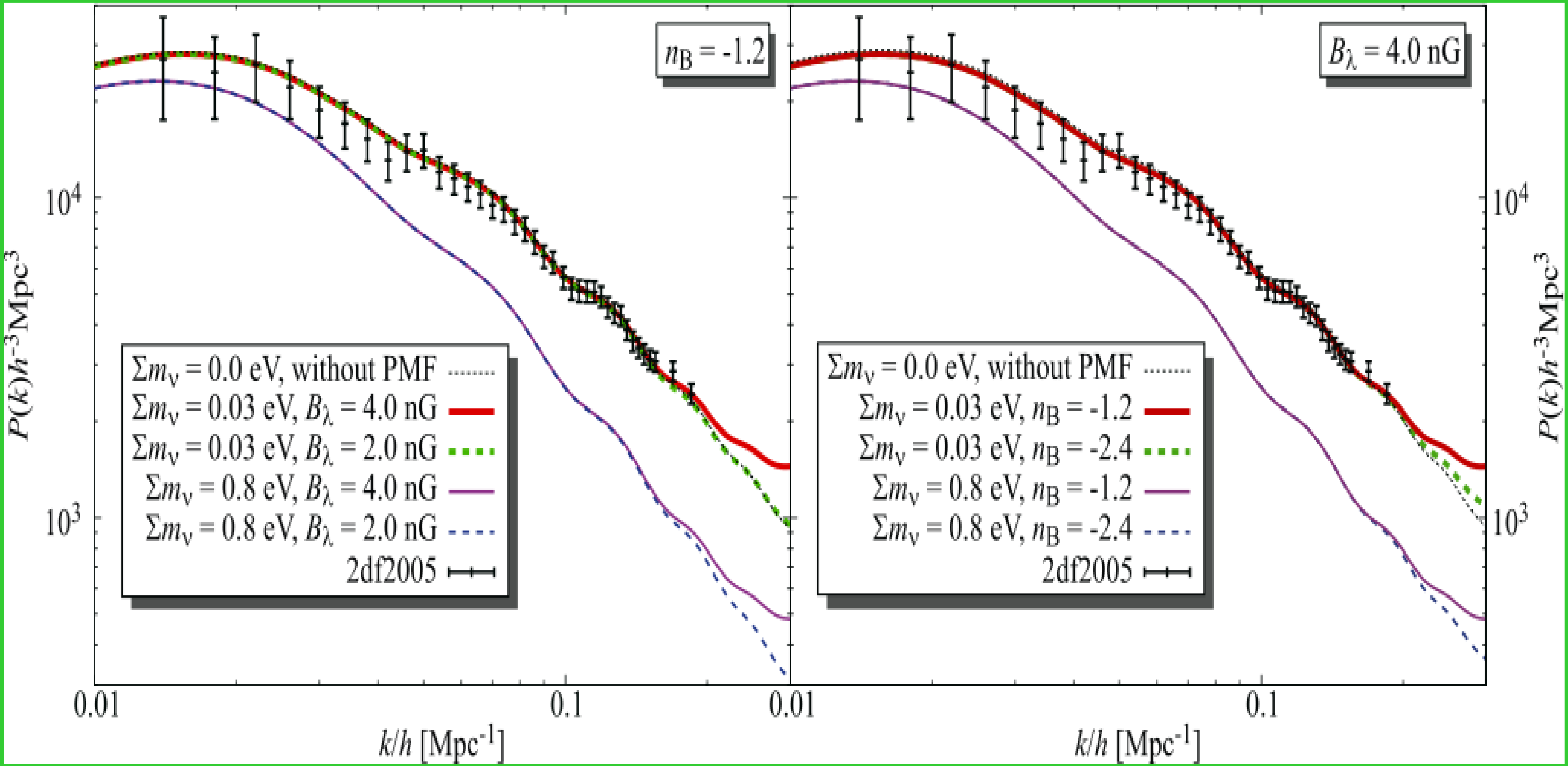
Massless or Massive is critical !

Cosmological constraints should be made carefully !

$$\underbrace{\Omega_b,\ \Omega_{CDM},\ \Omega_\Lambda,\ H_0,\ \tau,\ n_S,\ A_S,\ A_T/A_S}_{\text{Standard Cosmological Parameters}} \quad + \quad \underbrace{B,\ n_B}_{\text{Primordial Magnetic Field}} \quad + \quad \underbrace{m_\nu}_{\text{Neutrino Mass}}$$

Effects of a Neutrino Mass

m_ν affects matter power spectrum differently from PMF.

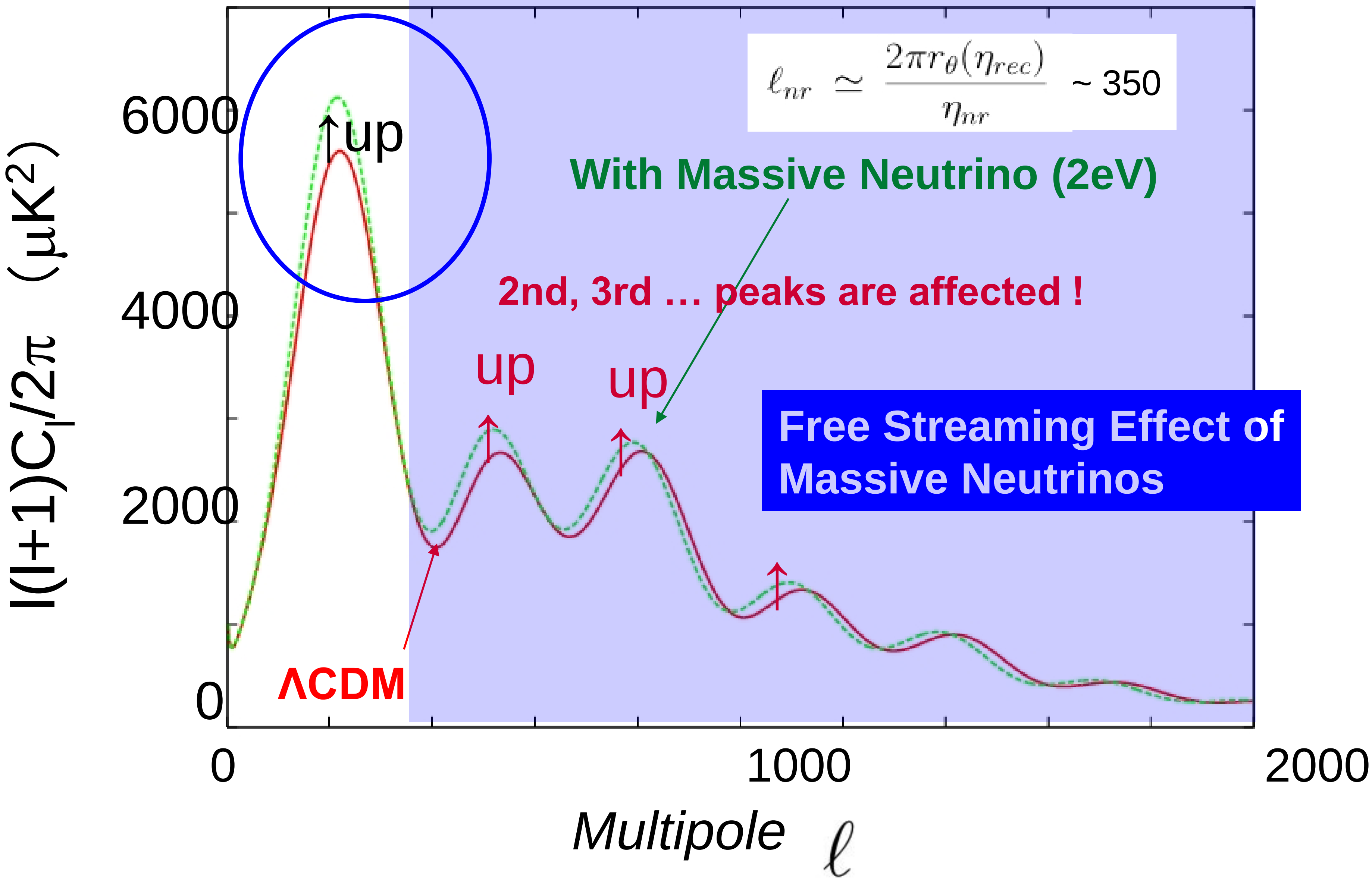


Effects of Neutrino Mass m_ν

$\vec{B}_{PMF} = \vec{0}$

Integrated Sachs-Wolfe Effect, similarly to CDM

S. Dodelson, E. Gates and A. Stebbins (1996)



Neutrino Mass Effect

☆**Analytical solution of massless neutrino anisotropic stress:**

$$\pi_\nu = -\pi_{PMF}\frac{R_\gamma}{R_\nu}\left(1-\frac{c(k\tau)^2}{4R_\nu+15}\right)$$

(Super horizon scale)

☆**Anisotropic stress of massive neutrino:**

$$\pi_h^{(m)} \simeq \pi_{PMF}^{(m)}\left(1-\frac{1}{2}\frac{5}{7\pi^2}H_0^2\Omega_R m_\nu^2\tau^2\right)$$

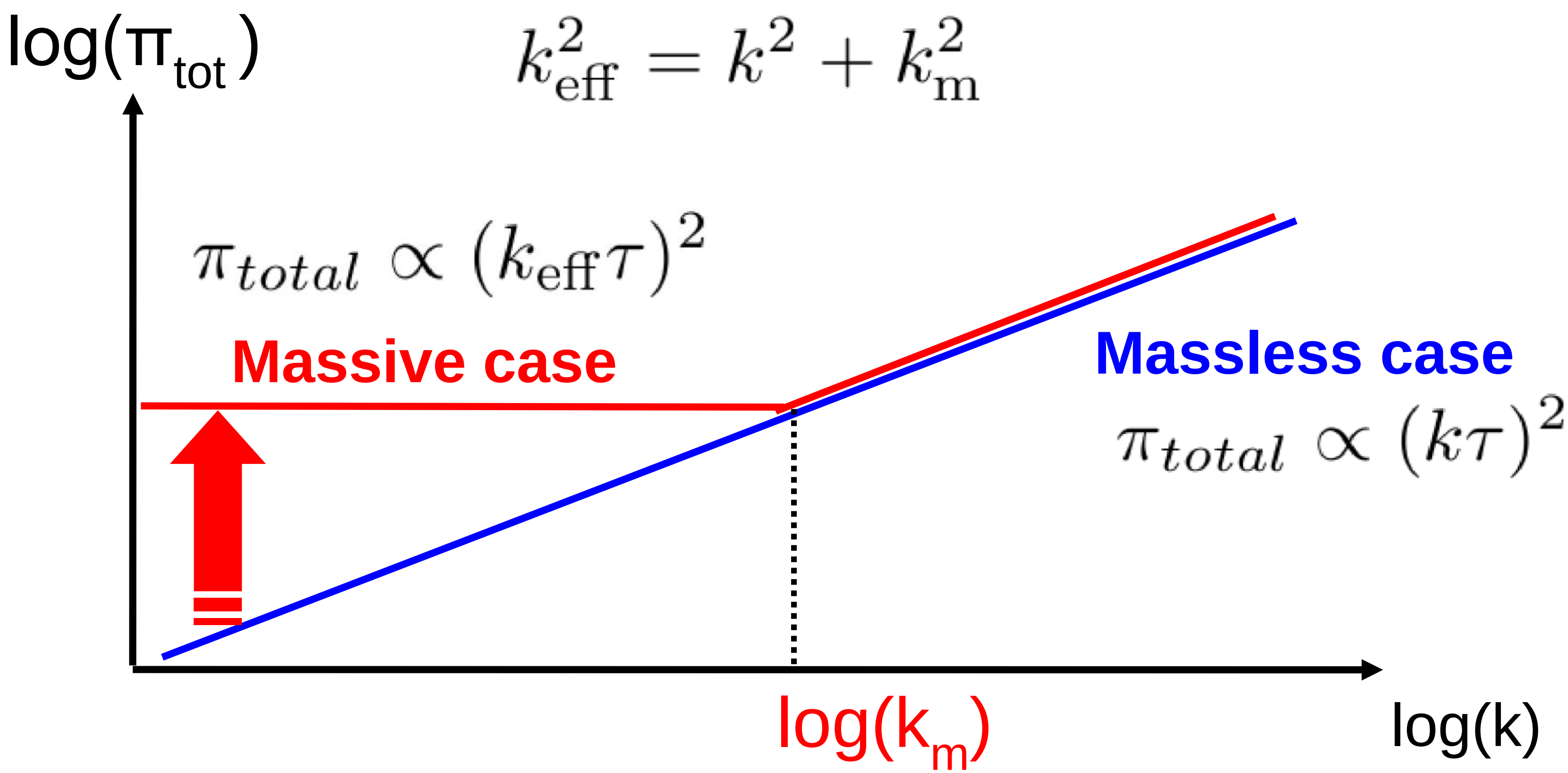
$$\simeq -\pi_{PMF}\frac{R_\gamma}{R_\nu}\left(1-\frac{c(k_{\rm eff}\tau)^2}{4R_\nu+15}\right)$$

Effective wave number

$$k_{\rm eff}^2 = k^2 + k_{\rm m}^2$$

$$k_{\rm m} = \sqrt{\frac{1}{2}\frac{5}{7\pi^2}H_0^2\Omega_R\frac{4R_\nu+15}{c}m_\nu^2}$$

Total anisotropic stress

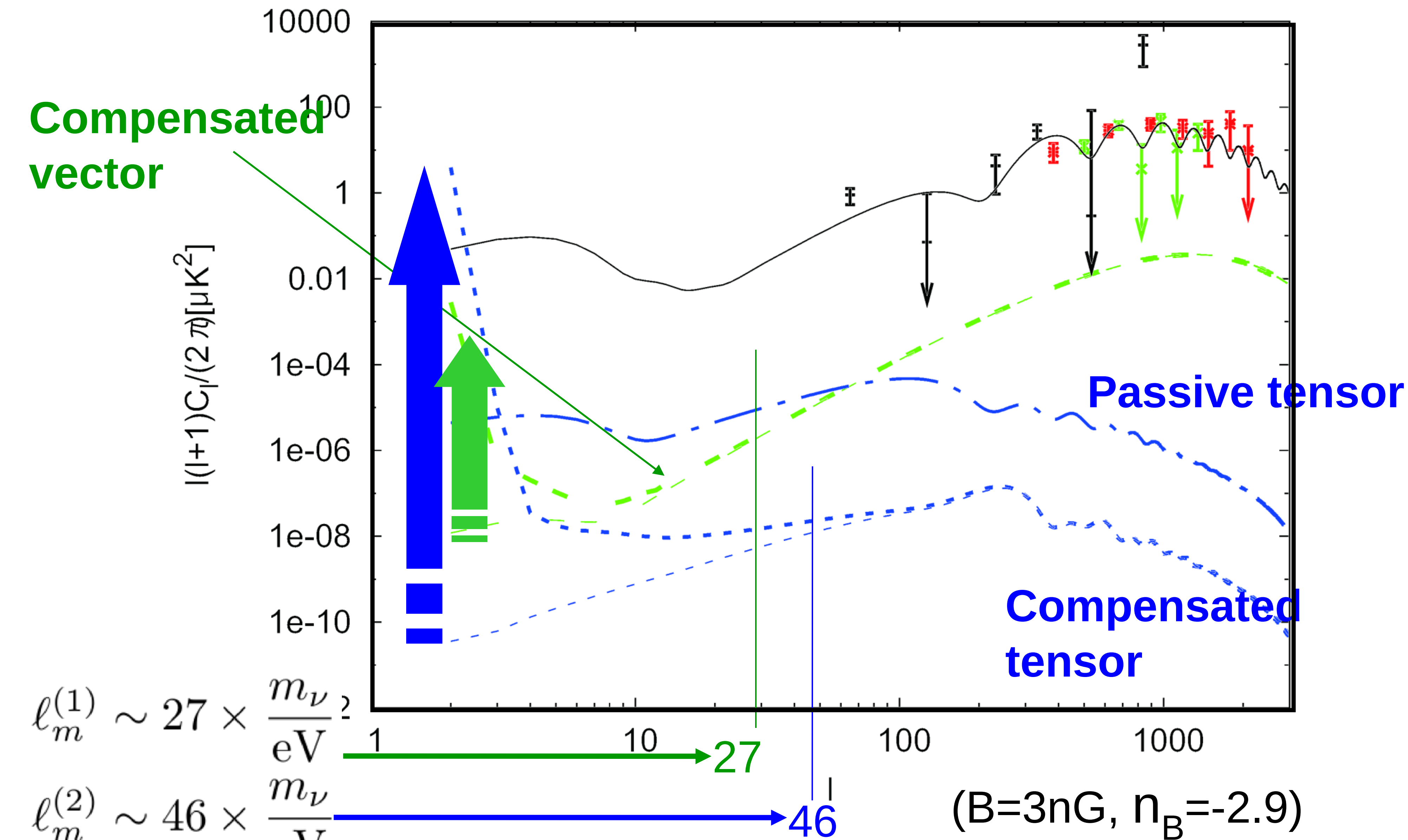


Vector mode $k_m^{(1)} = 1.9 \times 10^{-3} \times \frac{m_\nu}{\rm eV}\ {\rm Mpc}^{-1}$ $\ell_m^{(1)} \sim 27 \times \frac{m_\nu}{\rm eV}$

Tensor mode $k_m^{(2)} = 3.3 \times 10^{-3} \times \frac{m_\nu}{\rm eV}\ {\rm Mpc}^{-1}$ $\ell_m^{(2)} \sim 46 \times \frac{m_\nu}{\rm eV}$

EE Mode (m_ν=1eV)

Kojima, Ichiki, Yamazaki,k Kajino & Mathews, Phys. Rev. D78 (2008), 045010.



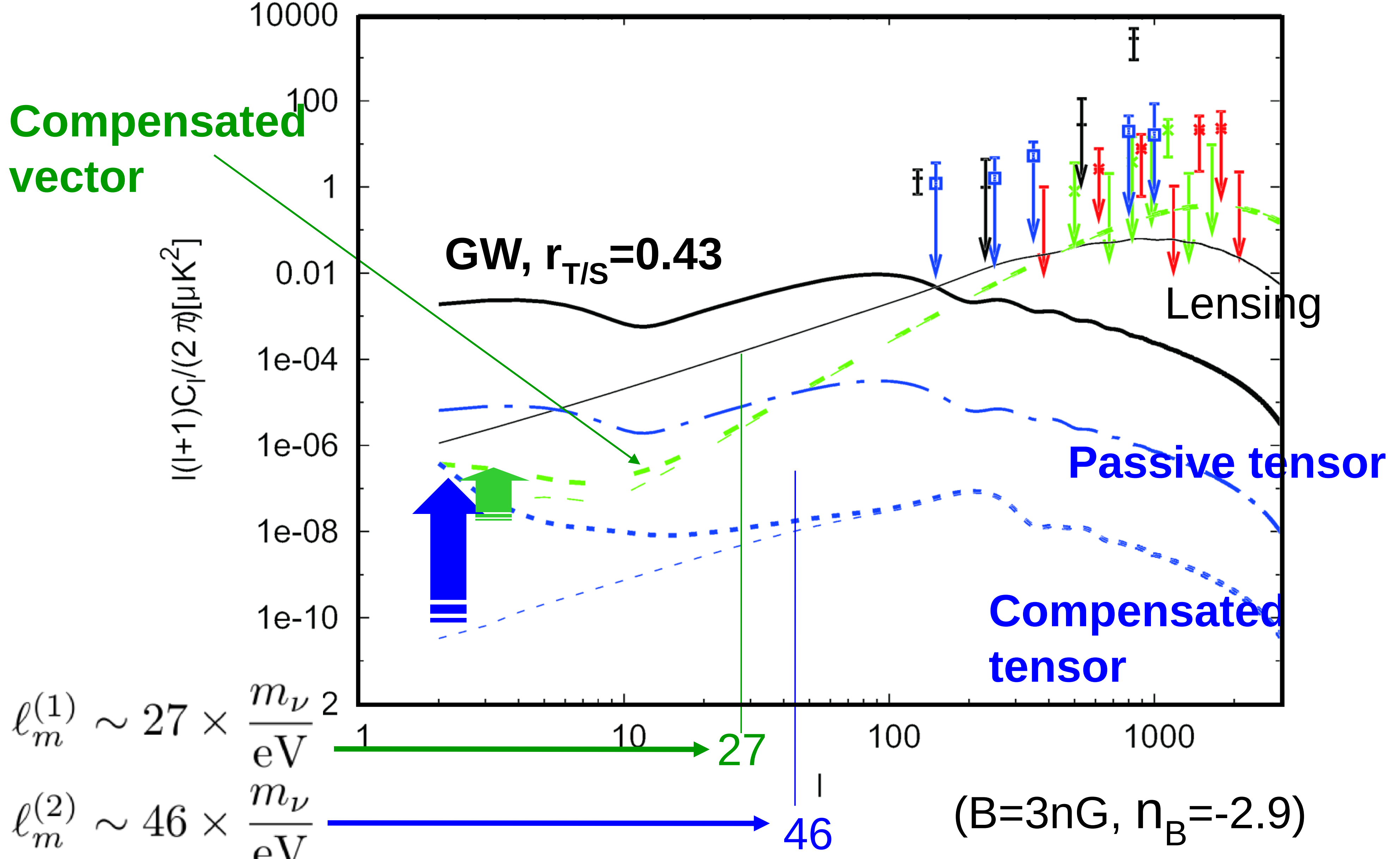
$$\ell_m^{(1)} \sim 27 \times \frac{m_\nu}{\mathrm{eV}}$$

$$\ell_m^{(2)} \sim 46 \times \frac{m_\nu}{\mathrm{eV}}$$

(B=3nG, n_B=-2.9)

BB Mode (m_ν=1eV)

Kojima, Ichiki, Yamazaki,k Kajino & Mathews, Phys. Rev. D78 (2008), 045010.

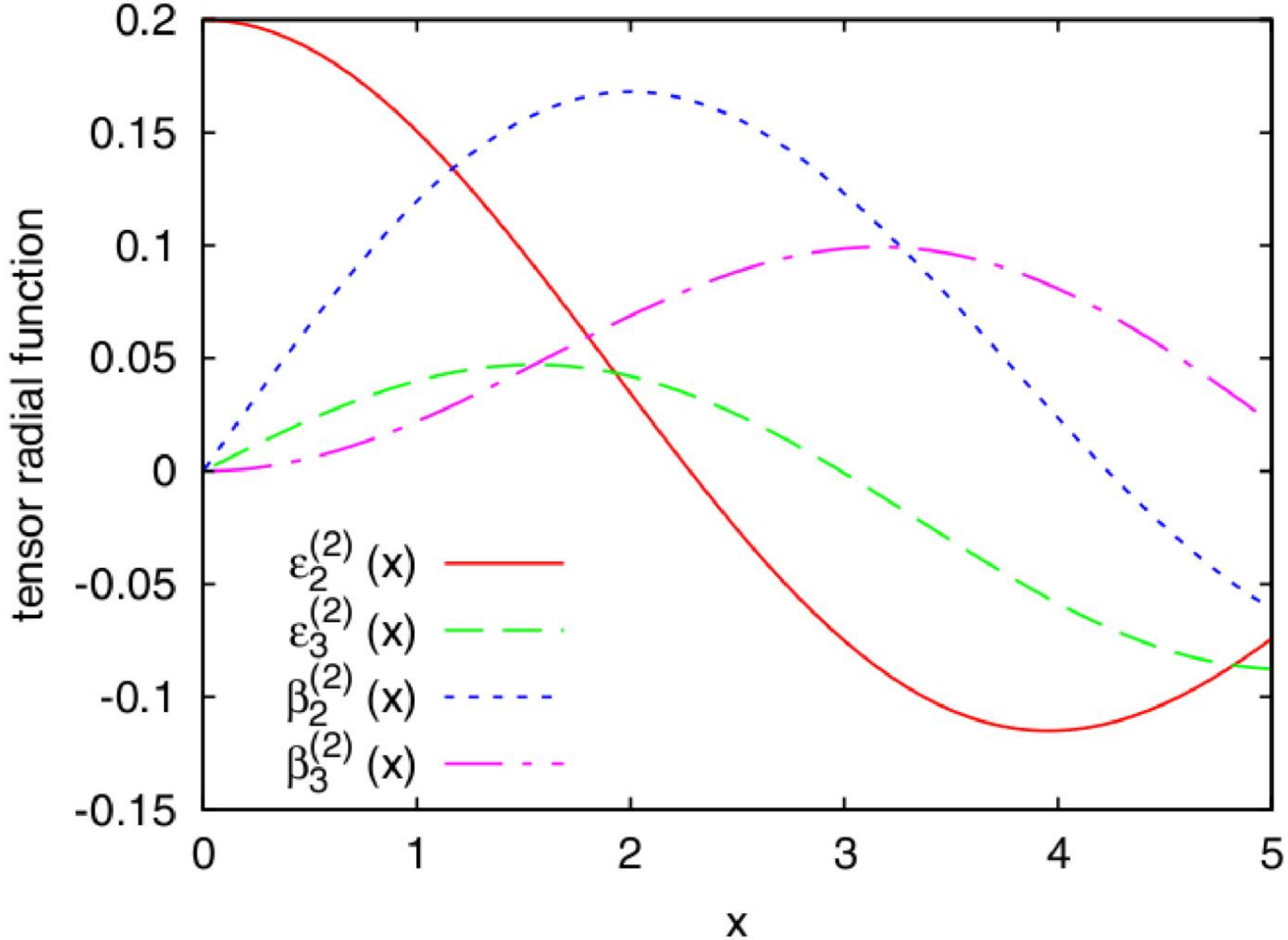


$$\ell_m^{(1)} \sim 27 \times \frac{m_\nu}{\mathrm{eV}}^2$$

$$\ell_m^{(2)} \sim 46 \times \frac{m_\nu}{\mathrm{eV}}$$

(B=3nG, n_B=-2.9)

Radial function



$$(2l+1)^2 C_l^{\mathrm{EE}(m)} \propto \int \frac{dk}{k} k^{2n_B+6} k_{\mathrm{eff}}^4 \epsilon_l^{(m)2}(k(\eta_0 - \eta_{\mathrm{rec}})).$$

Summary

1. We found that a large curvature fluctuation is generated from an extra-anisotropic stress so that the observed CMB and polarization are explained very well.

   This resolves a difficulty of too bluer power spectrum in Ekpyrotic or Cyclic model of brane world cosmology.

   However, this model cannot be an alternative to inflation because we need a scale-free source spectrum.

2. Best fit primordial magnetic field is constrained to be:

$$B = 0.85^{+1.25}_{-0.85} \text{ nG (@1 Mpc)} \qquad n_B = -2.37^{+0.88}_{-0.73}$$

3. Neutrino mass affects the CMB polarization, especially EE and BB modes, in the existence of primordial magnetic field.