

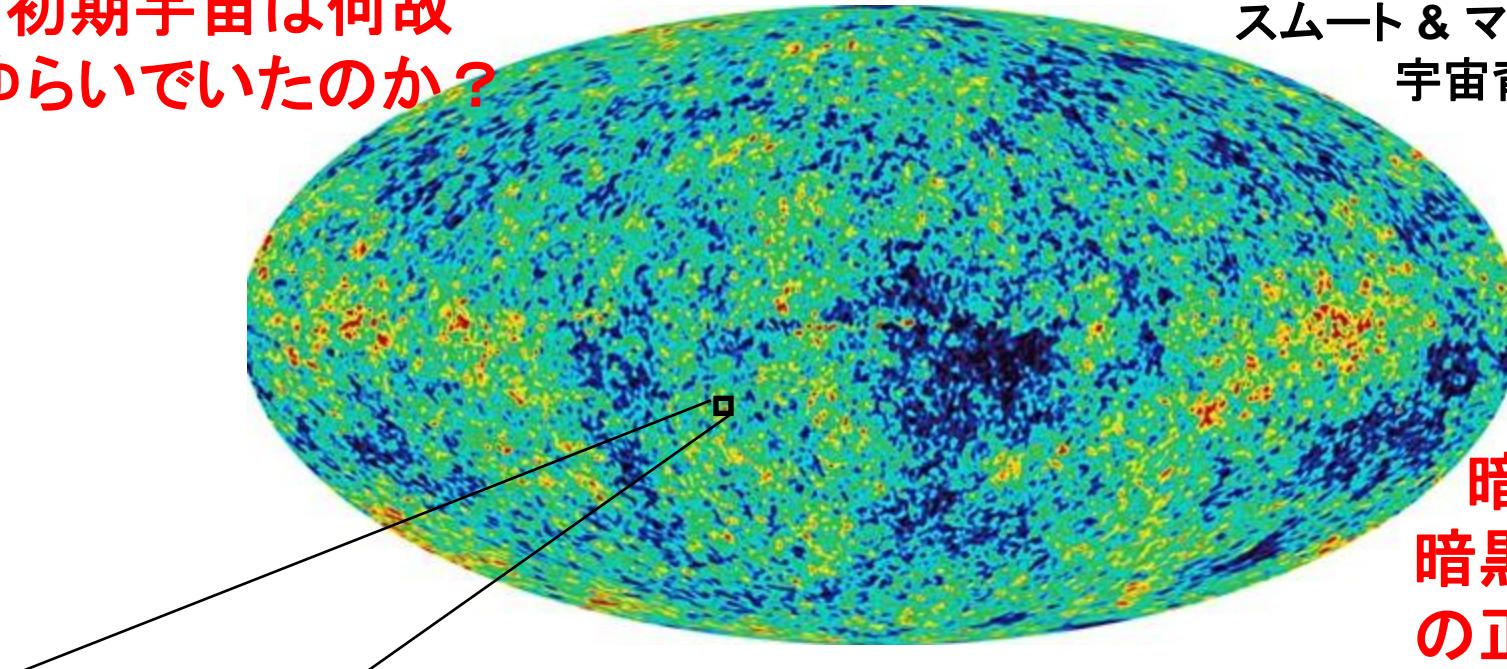
(2) ビッグバン元素合成と暗黒物質 SUSY 粒子モデル

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初期宇宙は何故
ゆらいでいたのか？

スムート & マザーが発見した
宇宙背景放射ゆらぎ



暗黒物質と
暗黒エネルギー
の正体は何か？

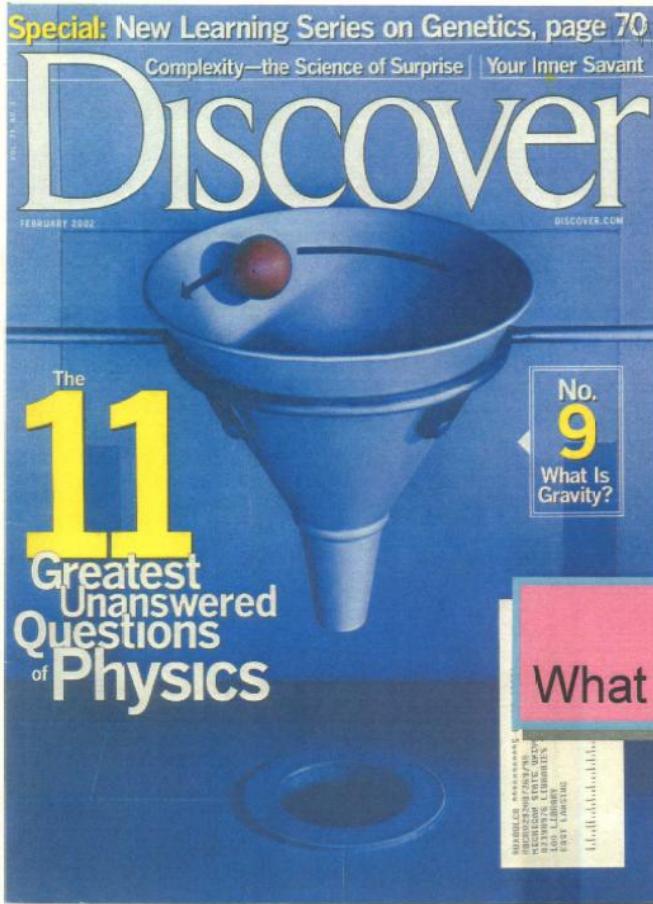


38万年 → 137億年

スーパーコンピュータを用いた宇宙構造形成過程のシミュレーション。
未知の**暗黒エネルギー73%**と**暗黒物質23%**を仮定すると、観測されている
現在の宇宙構造がうまく再現できる。

目的

「SUSY素粒子 Gravitino=暗黒物質」仮説を
ビッグバン元素合成で検証する。



**U.S. National Academy of
Science Report “Connecting
Quarks with the Cosmos:
Eleven Science Questions
for the New Century”**

Question 1
What is the dark matter ?

George Gamow's predictions in 1948



George Gamow

1. If the Universe began from the Fire Ball of hot Big-Bang and then expanded, we can detect today the **Cosmic Background Radiation of $T = 5 \text{ K}$!**
 - **2.7K CBR was discovered by Penzias & Wilson (1965)**
 - **CMB anisotropies by Smoot & Mathar (1992)**
2. In the early hot Big-Bang Universe were created (hopefully) almost all **atomic nuclides !**
 $^4\text{He} \text{ & } ^7\text{Li}$, discovered by astronomers (1980')

Observational Signature



Big-Bang Nucleosynthesis !

The Power of BBN is that the Physics is Accessible

Thermodynamic Equilibrium of Particles and Nuclei

$$n_i(p)dp = \frac{1}{2\pi^2}g_i p^2 \left[\exp\left(\frac{E_i(p) - \mu_i}{kT}\right) \pm 1 \right]^{-1} dp$$

$$\rho_i = \int p [n_i(p) + n_{\bar{i}}(p)] dp$$

$$\rho_\gamma = \frac{\pi^2}{15}(kT_\gamma)^4 , \quad \rho_{\nu_i} = \frac{7}{8}\frac{\pi^2}{15}(kT_\nu)^4$$

$$\rho = \rho_\gamma + \rho_{\nu_i} + \rho_i = \frac{\pi^2}{30}g_{eff}(kT)^4$$

$$g_{eff}(T) = \sum_{\text{bose}} g_{\text{bose}} + \frac{7}{8} \sum_{\text{fermi}} g_{\text{fermi}}$$

Cosmic Expansion

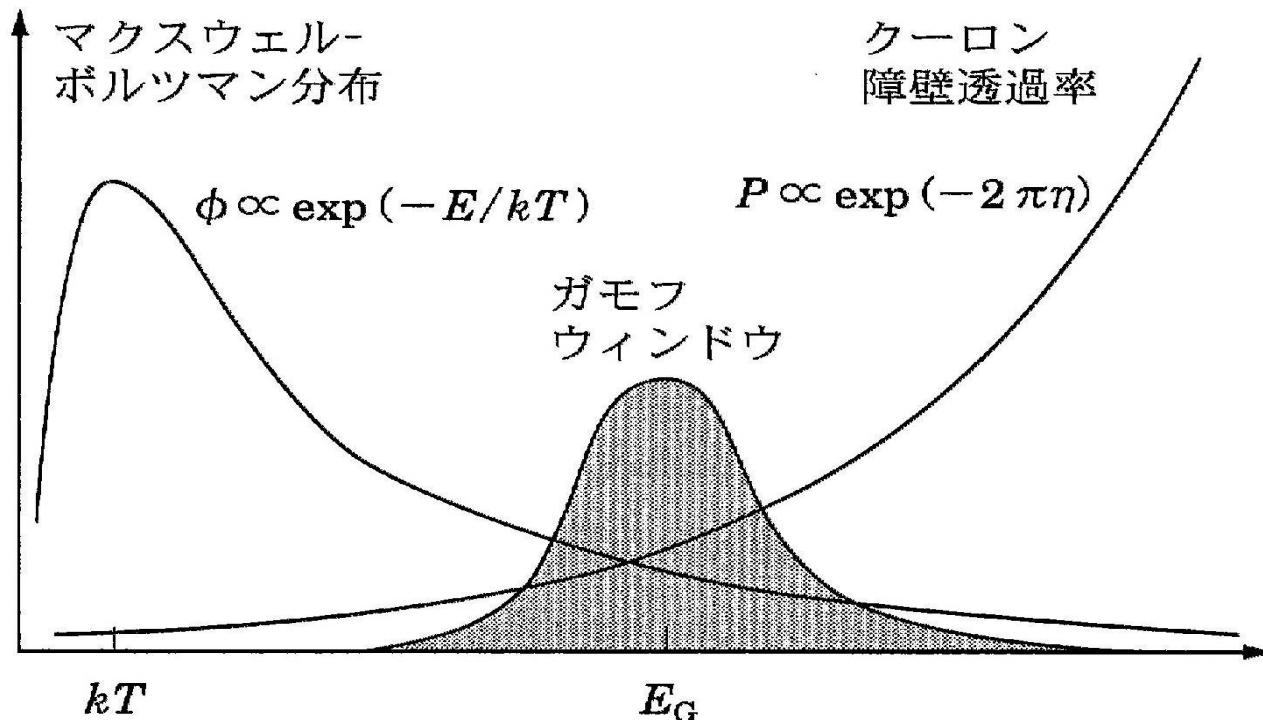
$$H^2(t) = \left(\frac{1}{R} \frac{dR}{dt} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{R^2}$$

Nuclear Reactions

$$\frac{dY_i}{dt} = \sum_{ijk} N_i \left(\frac{Y_l^{N_l} Y_k^{N_k}}{N_l! N_k!} \langle n_k \sigma_{lk} v \rangle - \frac{Y_i^{N_i} Y_j^{N_j}}{N_i! N_k!} \langle n_j \sigma_{ij} v \rangle \right)$$

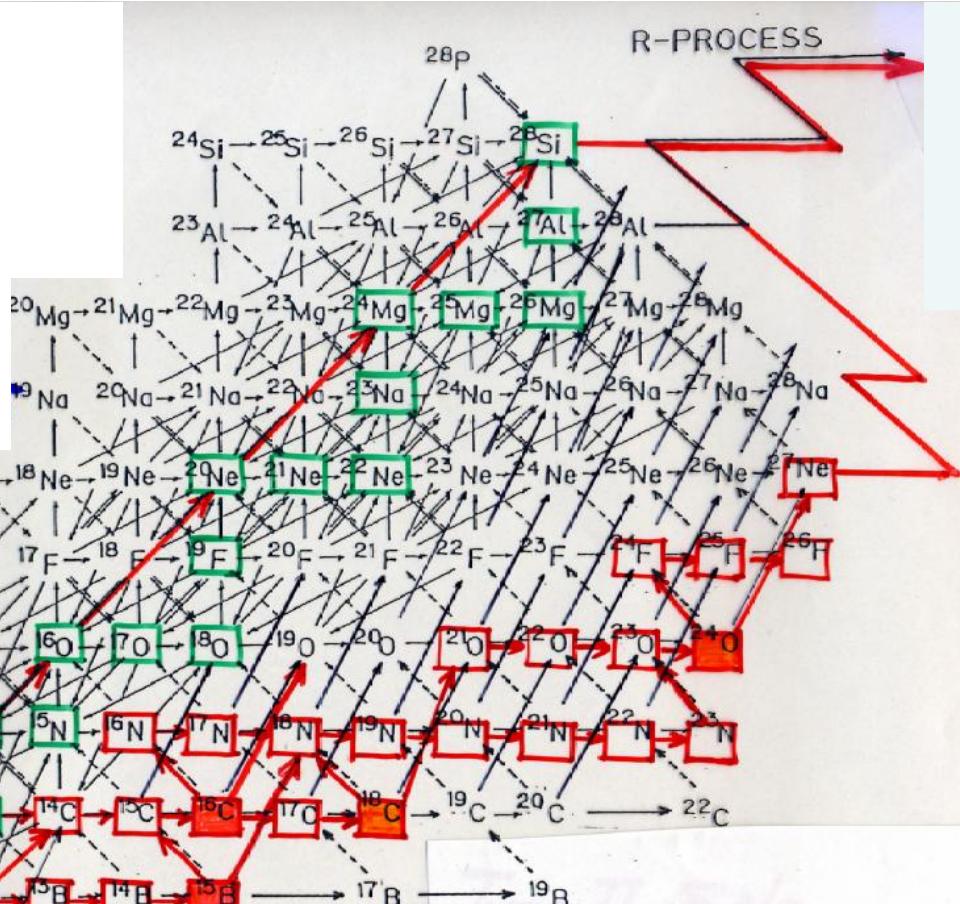
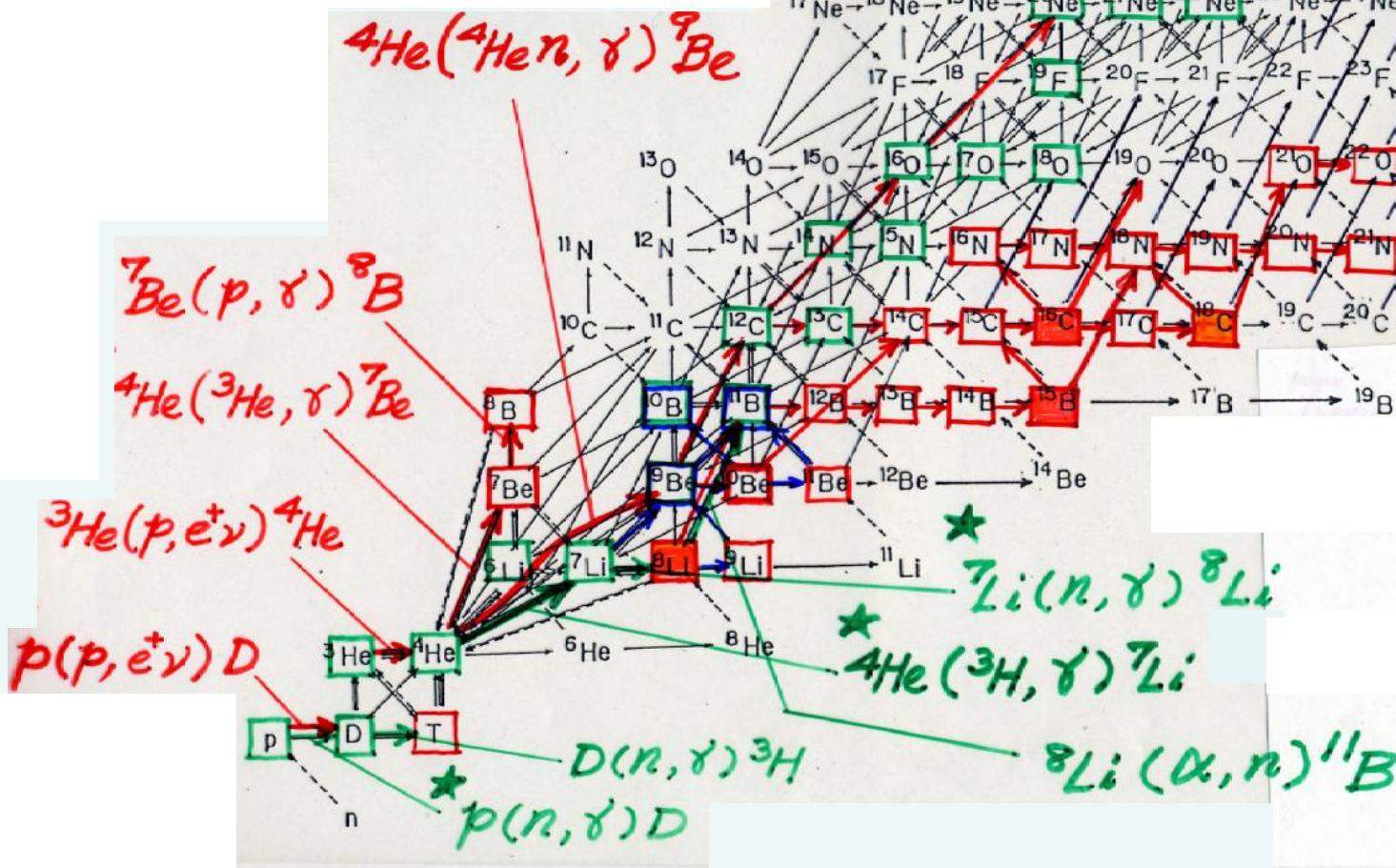
$$\sigma(E) = \frac{S(E)}{E} \exp(-2\pi\eta)$$

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int S(E) \exp(-2\pi\eta) \exp(-E/kT) dE$$

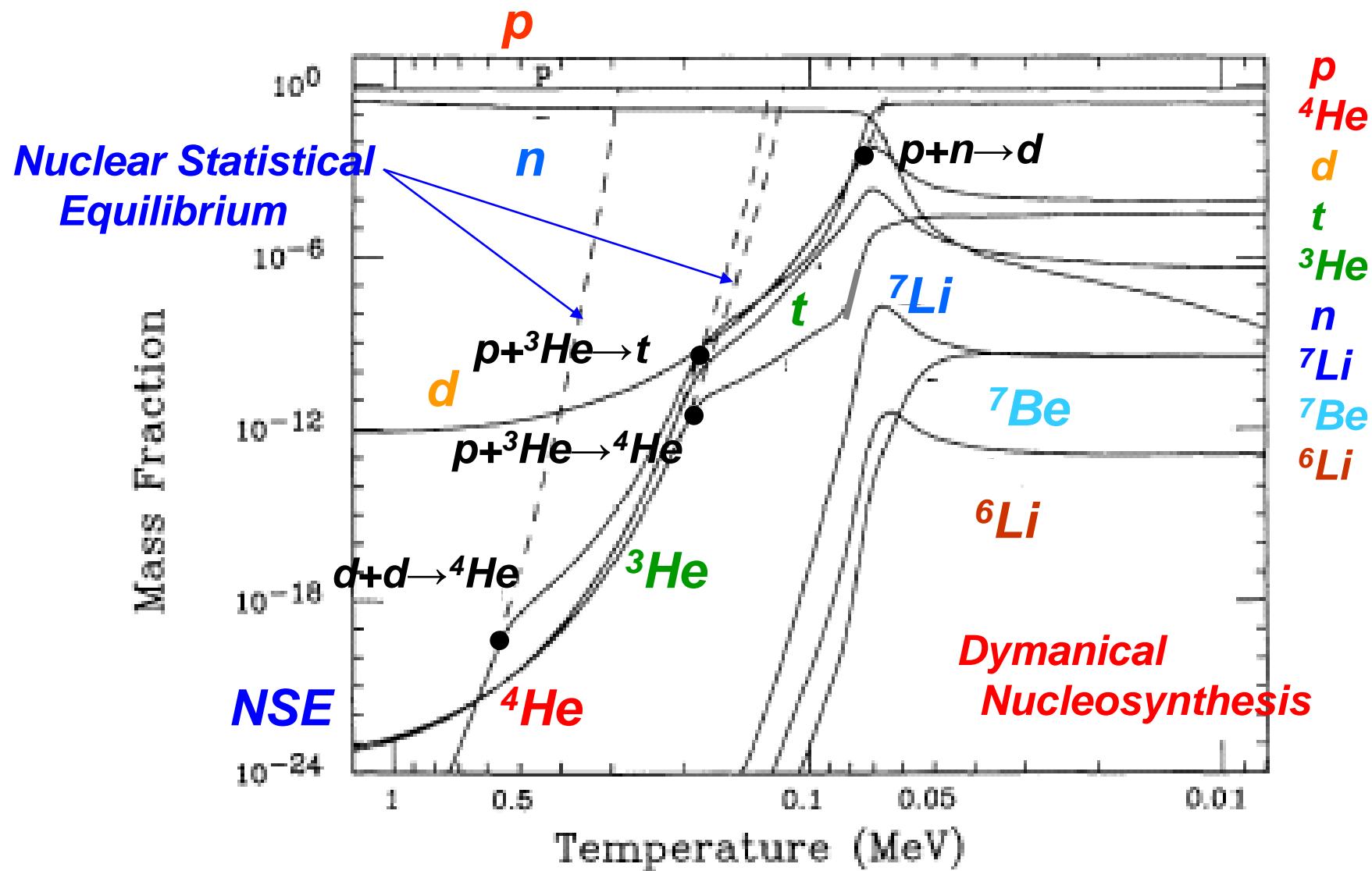


$$E_G = \left(\frac{\mu}{2} \right)^{1/3} \left(\frac{\pi Z_A Z_a e^2 k T}{\hbar} \right)^{2/3}$$

Reaction Network for Big-Bang Nucleosynthesis



Evolution of Abundances



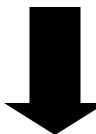
Nuclear Statistical Equilibrium (NSE)



nuclear mass A is the sum
of protons and neutrons $A = Z + N$

$$Z \mu_p + N \mu_n = \mu_A + \mu_\gamma (=0)$$

$$Y_A = n_A / n_B = g_A (m_A T / 2\pi)^{3/2} \exp\{(\mu_A - m_A)/T\} / n_B$$



Binding Energy
of Nucleus A

NSE Equation

$$Y_{A(Z,N)} \approx [S^{1-A}] G \pi^{\frac{3}{2}(A-1)} 2^{\frac{1}{2}(A-3)} A^{3/2} \left(\frac{T}{m_b} \right)^{\frac{3}{2}(A-1)} Y_p^Z Y_n^N e^{Q_A/T}$$

Entropy/Baryon: $S = 10^{+9}$ (in Big-Bang), 10^{+2} (in Supernovae)

H, D, $^{3,4}\text{He}$, $^{6,7}\text{Li}$

Fe-Co-Ni

Beryon-to-Photon Ratio, $\eta = n_B/n_\gamma$, vs. Entropy per Baryon, S/k

Entropy density = s

$$s = 2\pi^2/45 \cdot g^* \cdot T^3 + s(\text{NR})$$

$$g^* = \sum_{\text{Bosons}} g_B (T_B/T)^4 + \frac{7}{8} \sum_{\text{Fermions}} g_F (T_F/T)^4$$

Photon number density = n_γ

$$n_\gamma = \zeta(3)/\pi^2 \cdot g_\gamma \cdot T^3$$

Entropy per Baryon = S/k

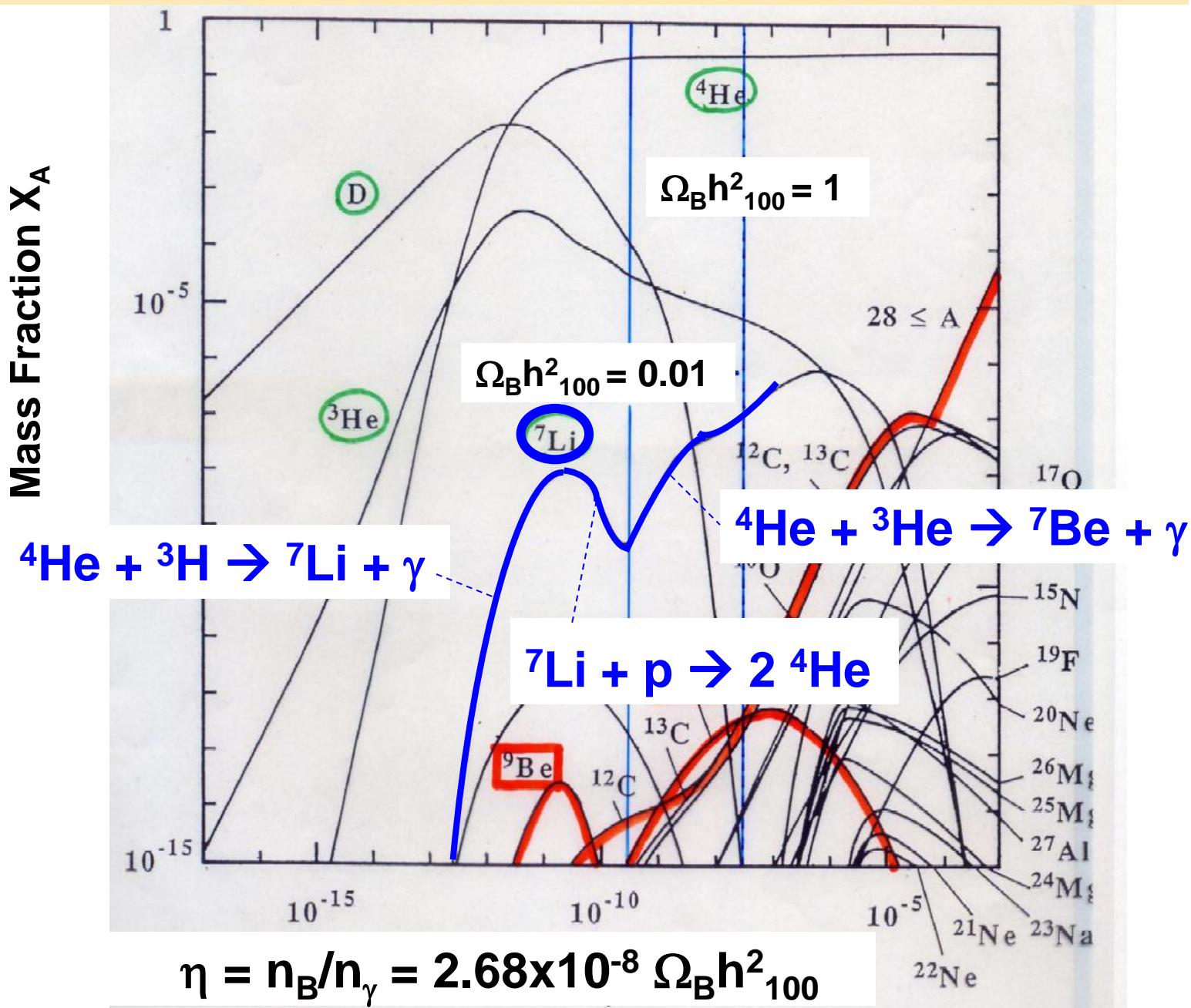
$$S/k = s_\gamma/n_B \doteq 3.6 n_\gamma/n_B = 3.6 \eta^{-1}$$

Early Universe 10^9

$$\eta = n_B/n_\gamma = 10^{-9}$$

$$\Omega_B h^2 = 3.73 \times 10^7 \eta$$

Big-Bang (Primordial) Nucleosynthesis



Big-Bang Nucleosynthesis

D, ^3He , ^4He and ^7Li

Y_p - Extragalactic HII Regions

$$0.240 \leq Y_p \leq 0.244$$

Izotov & Thuan (2003)

$$0.240 \leq Y_p \leq 0.258$$

Olive & Skillman (2004)

D - QSO absorption systems

$$2.4 \times 10^{-5} \leq D/H \leq 3.2 \times 10^{-5}$$

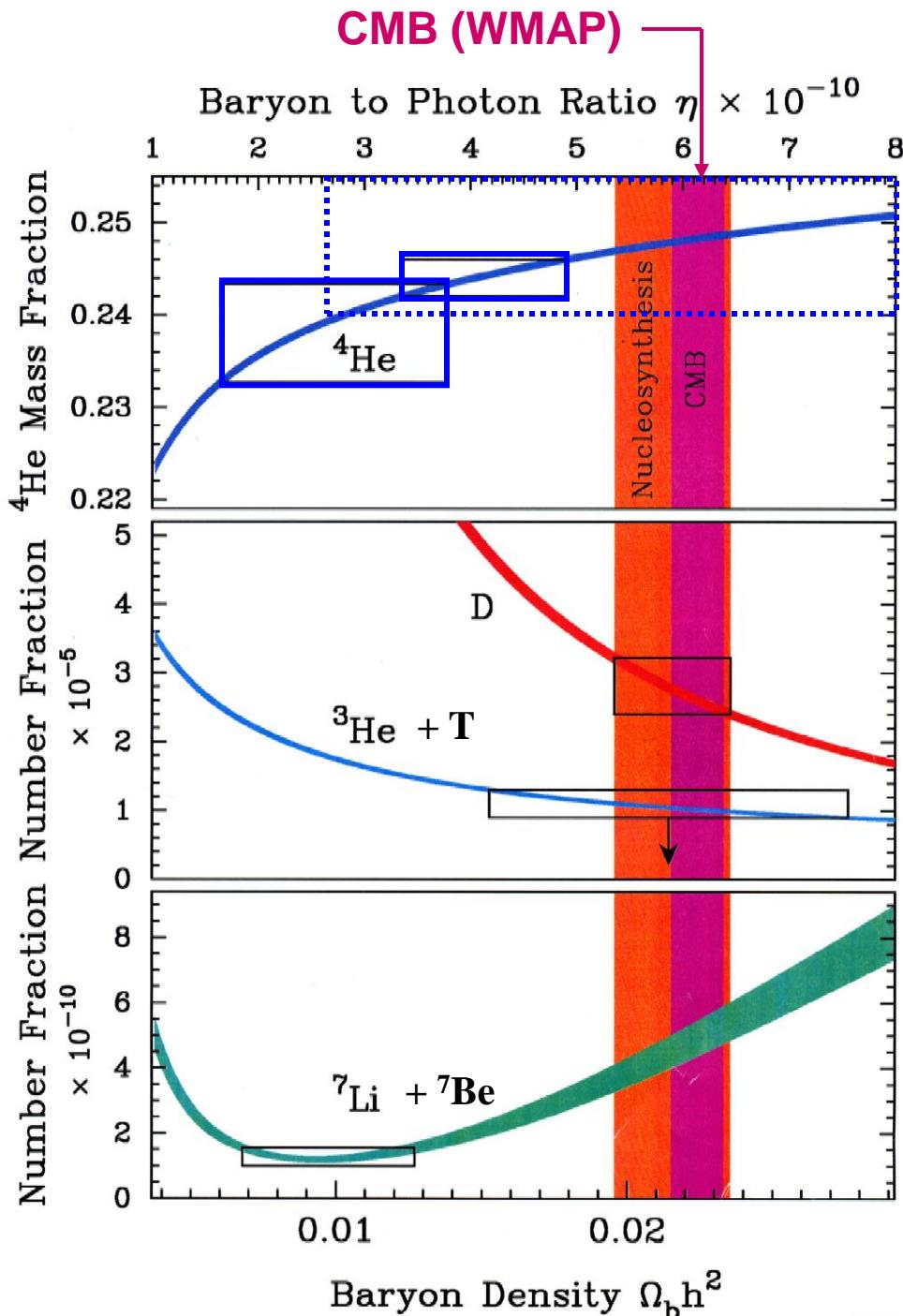
Kirkman et al. (2003)

WMAP

^7Li - Halo Stars

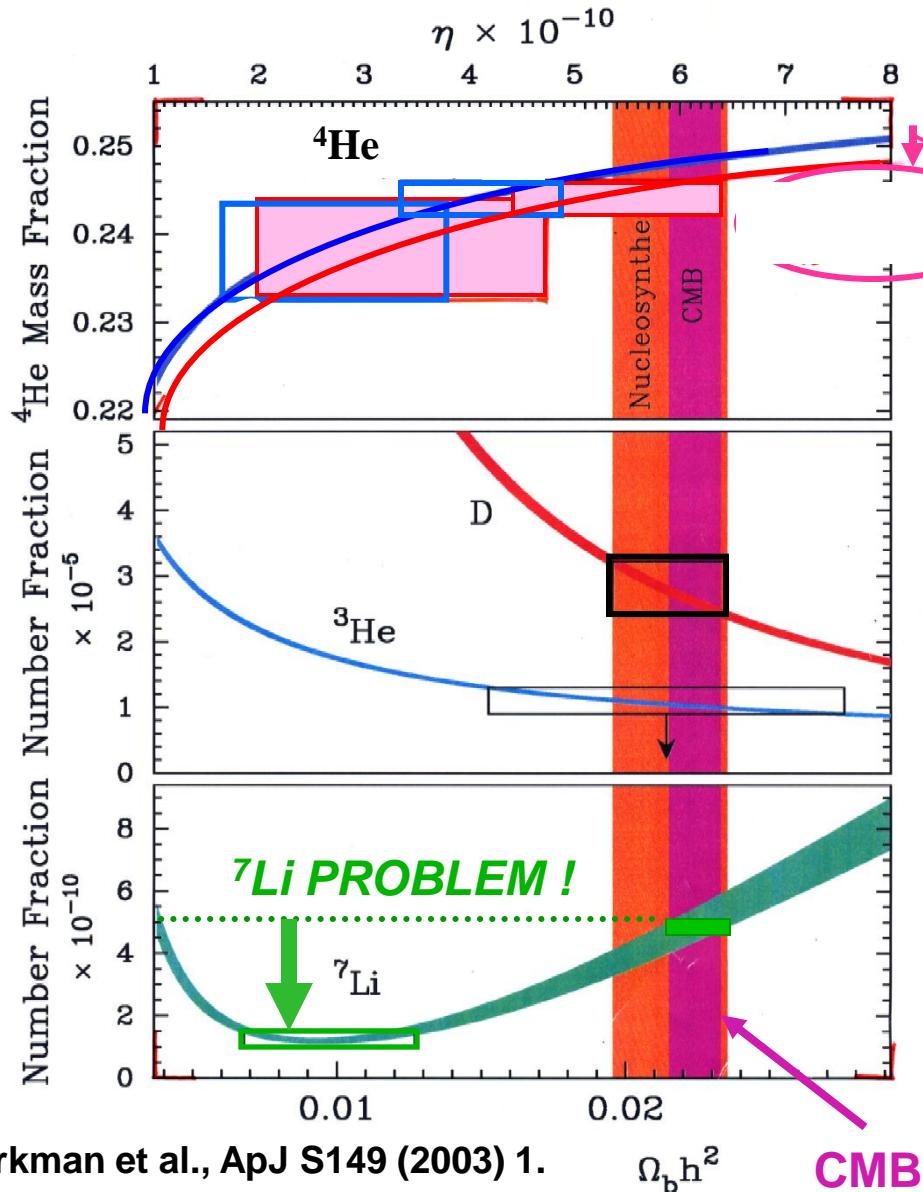
$$0.91 \times 10^{-10} \leq ^7\text{Li}/H \leq 1.91 \times 10^{-10}$$

Ryan et al. (2000)



BBN and Particle Physics

Smith, Kawano & Malaney, ApJ S85(2003) 219; Mathews, Kajino & Shima, PRD71 (2005) 21302 (R).



Effect of Neutro-Life on BBN- ${}^4\text{He}$: $2\text{p} + 2\text{n} \longrightarrow {}^4\text{He}$

Boltzmann distribution

$$n_A = g_A \left(\frac{m_A T}{2\pi k} \right)^{\frac{3}{2}} \exp \left(-\frac{\mu_A - m_A}{T} \right)$$

Weak Equilibrium until $T \approx T_d$ (Decoupling Temp.)



$$\underset{\substack{\downarrow \\ 0}}{\mu_p + \mu_e} = \mu_n + \mu_{\gamma_e}$$

$$\boxed{n \equiv \frac{n_n}{n_p} = \left(\frac{m_n}{m_p} \right)^{\frac{3}{2}} \exp \left(-\frac{\Delta m}{T_d} - \frac{\mu_b}{T_d} \right) < 1.} \quad (1)$$

${}^4\text{He}$ Synthesis ($2\text{p} + 2\text{n} \rightarrow {}^4\text{He}$)

Approximation: All neutrons ($n_n < n_p$) are interconverted to ${}^4\text{He}$, and nucleosynthesis quite suddenly when $n_n \approx 0$.

$$\boxed{\gamma_p \equiv \frac{4n_n}{2n_p + 2n_n} = \frac{\bar{n}_n}{1 + \bar{n}}.} \quad (2)$$

Weak Decoupling Temperature

$$\boxed{T_d^3 = \frac{1.66}{M_p e} \times \frac{\sqrt{g_F}}{G_F^2}.} \quad (3)$$

1st effect: $\delta \tau_n \rightarrow \delta T_d \rightarrow \delta(n/p) \rightarrow \delta({}^4\text{He})$

$$(1); \quad \delta \tau_n = \frac{\Delta m}{T_d^2} e^{-\frac{\Delta m}{T_d}} \delta T_d = \frac{\Delta m}{T_d^2} \tau_n \delta T_d$$

$$(2); \quad \delta \gamma_p = \frac{\bar{n}}{1 + \bar{n}} \delta \tau_n - \frac{2\bar{n}}{(1 + \bar{n})^2} \delta \tau_n = \frac{\bar{n}}{1 + \bar{n}} \left(1 - \frac{\bar{n}}{2} \right) \delta \tau_n$$

$$\therefore \delta \gamma_p = \gamma_p \left(1 - \frac{\bar{n}}{2} \right) \left(\frac{\Delta m}{T_d} \right) \left(\frac{\delta T_d}{T_d} \right). \quad (4)$$

$$\boxed{T_n^{-1} = \Gamma_{n \rightarrow p + \gamma_e} = \frac{G_F^2}{2\pi^3} (1 + 3g_A^2) M_p^5 \int_0^\infty d\epsilon \epsilon (\epsilon - \bar{p})^2 (\epsilon^2 - 1)^{\frac{1}{2}}}$$

$(g_A \approx 1.26) \approx 1.636$

Axial-vector coupl.

$$\boxed{\tau_n \propto G_F^2}$$

$$\boxed{\frac{\delta \tau_n}{\tau_n} = (-x) \frac{\delta G_F}{G_F}}$$

$$(3); \quad 3 \frac{\delta T_d}{T_d} = (-x) \frac{\delta G_F}{G_F} = \frac{\delta T_n}{T_n}$$

$$(4); \quad \delta \gamma_p = \frac{1}{3} \gamma_p \left(1 - \frac{\bar{n}}{2} \right) \left(\frac{\Delta m}{T_d} \right) \left(\frac{\delta T_n}{T_n} \right) \rightarrow \boxed{\frac{(\delta \gamma_p)}{(\frac{\delta T_n}{T_n})} \approx 0.3 !}$$

2nd effect: $\delta \tau_n \rightarrow \delta n \rightarrow \delta({}^4\text{He})$

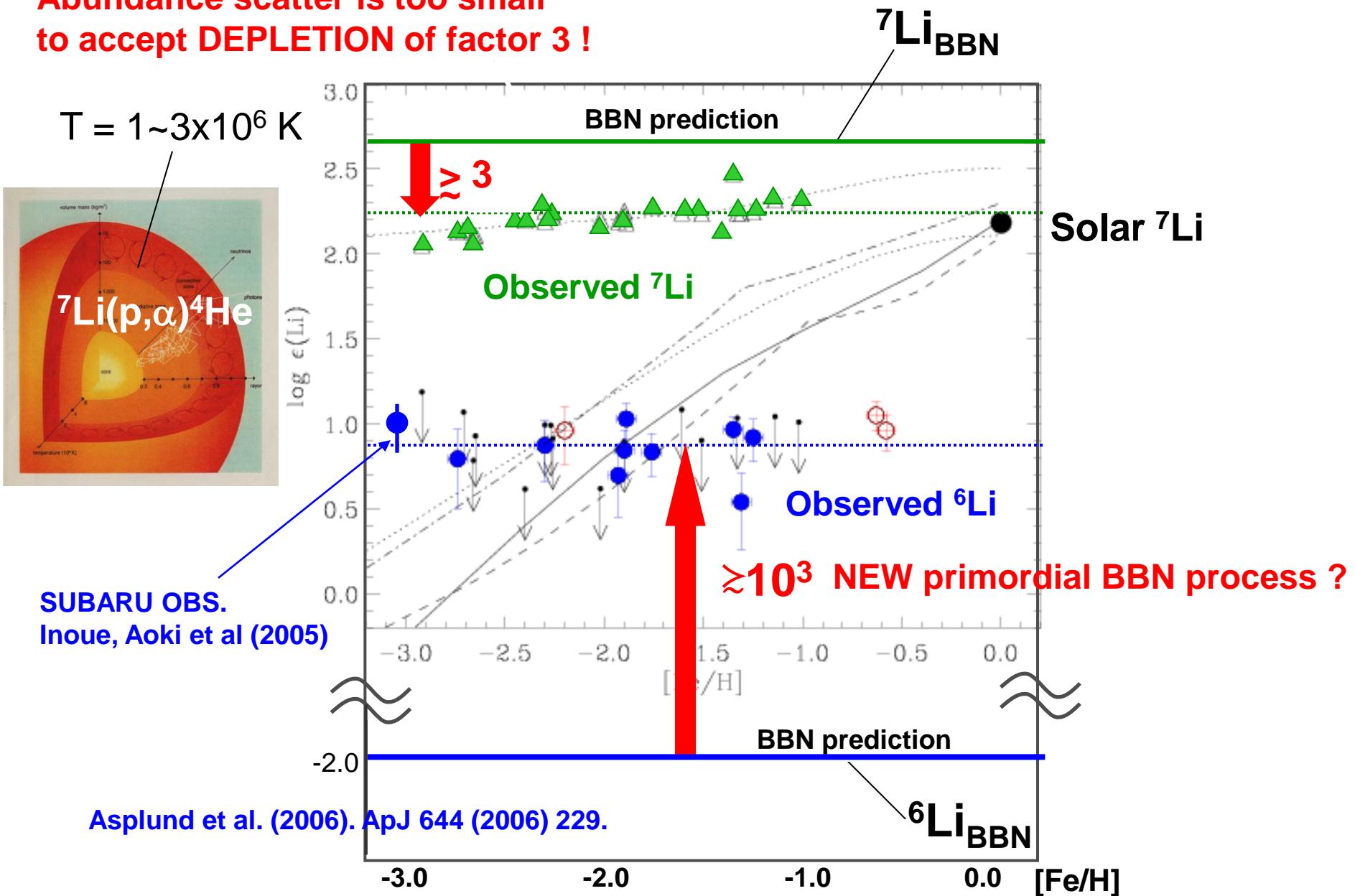
Freezeout time of $p + n \rightleftharpoons D + \gamma$ changes.

NET EFFECT:

$$\delta \tau_n < 0 \rightarrow \delta({}^4\text{He}) < 0$$

Plateau like HIGH $^{6,7}\text{Li}$ ABUNDANCE --- primordial ?

Abundance scatter is too small
to accept DEPLETION of factor 3 !



How to solve HIGH ${}^6\text{Li}$ primordial abundance?

Ellis et al. (1986); Moroi and Kawasaki (1994); Jedamzik PRL 84 (2000) 3248;
Cyburt et al., PRD 67 (2003) 103521; Ellis et al. PLB619 (2005) 30;
Kusakabe, Kajino & Mathews, D74 (2006), 023526, PR D76 (2007) 121302(R);
ApJ 680 (2008) 846

Cosmological Solution

1st possibility: SUSY Leptonic DM particles (Stau) X^{+-} are bound to ${}^4\text{He}$, ${}^7\text{Li}$, ${}^7\text{Be}$ and catalize new BBN:



2nd possibility: Decaying relic DM particles X decay to non-th γ 's:



SUSY-CDM Model

stau (scalar tau) ($\tilde{\tau}$) \Rightarrow **X⁺⁻**

- 1) Supersymmetry partner (boson) of the **tau**-lepton (fermion)
stau (spin = 0)
- 2) **Gravitino** = a candidate of the lightest SUSY particle (LSP)
(Fermion partner of the graviton, a candidate for **CDM**)
Then, the next lightest SUSY particle (NLSP) **stau decays!**
- 3) **lifetime** $\sim \tau_X > 10^3$ sec (decay to **gravitino**)
mass $\sim m_X$ of order 100 GeV
- 4) **charged lepton** ; Coulombic interaction and weak interaction

stau is expected to be discovered at **LHC**
at the early stage after the first beam.

A

Binding energy of A_x

nuclide	r_c^{RMS} (fm)	Reference	E_{Bind} (MeV)
${}^1\text{H}$	0.875 ± 0.007	Yao et al. (2006)	0.025
${}^2\text{H}$	2.116 ± 0.006	Simon et al. (1981)	0.049
${}^3\text{H}$	1.755 ± 0.086	TUNL Nuclear Data Evaluation	0.072
${}^3\text{He}$	1.959 ± 0.030	TUNL	0.268
${}^4\text{He} x$	1.80 ± 0.04	Tanihata et al. (1988)	0.343
${}^6\text{Li}$	2.48 ± 0.03	Tanihata	0.806
${}^7\text{Li}$	2.43 ± 0.02	Tanihata	0.882
${}^8\text{Li}$	2.42 ± 0.02	Tanihata	0.945
${}^6\text{Be}$	2.52 ± 0.02	We took ${}^7\text{Be}$ radius	1.234
${}^7\text{Be} x$	2.52 ± 0.02	Tanihata	1.324
${}^8\text{Be}$	2.52 ± 0.02	We took ${}^7\text{Be}$ radius	1.401
${}^9\text{Be}$	2.50 ± 0.01	Tanihata	1.477
${}^7\text{B}$	2.68 ± 0.12	We took ${}^8\text{B}$ radius	1.752
${}^8\text{B}$	2.68 ± 0.12	Fukuda et al. (1988)	1.840
${}^9\text{B}$	2.68 ± 0.12	We took ${}^8\text{B}$ radius	1.917

Binding energies of X-nuclei like ${}^4\text{He}_X$

Assumptions

- X- has spin 0, charge -e, mass $m_X \gg 1$ GeV
- Nuclides have Gaussian charge distributions.

$$\rho(r) = Ze(\pi r_0^2)^{-3/2} \exp(-r^2 / r_0^2)$$

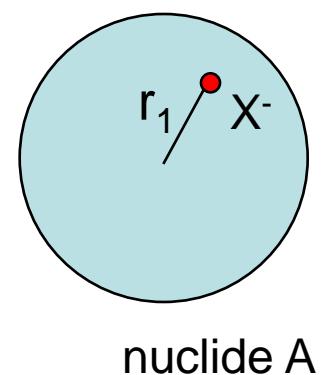
$$r_0^2 = \frac{2}{3} \langle r_c^2 \rangle$$

mean square charge radius

Two-body Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V(r) - E \right] \psi_{lm}(\mathbf{r}) = 0$$

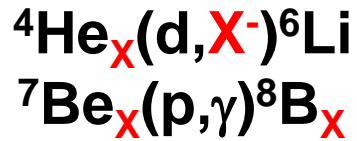
$$V(r_1) = \int_0^{r_1} \rho(r) \frac{e}{r_1} d^3r = \frac{Ze}{r_1} \operatorname{erf}(r_1 / r_0)$$



Gaussian expansion method, Kamimura, Kii & Hiyama et al. (2003)

Cosmological Solution

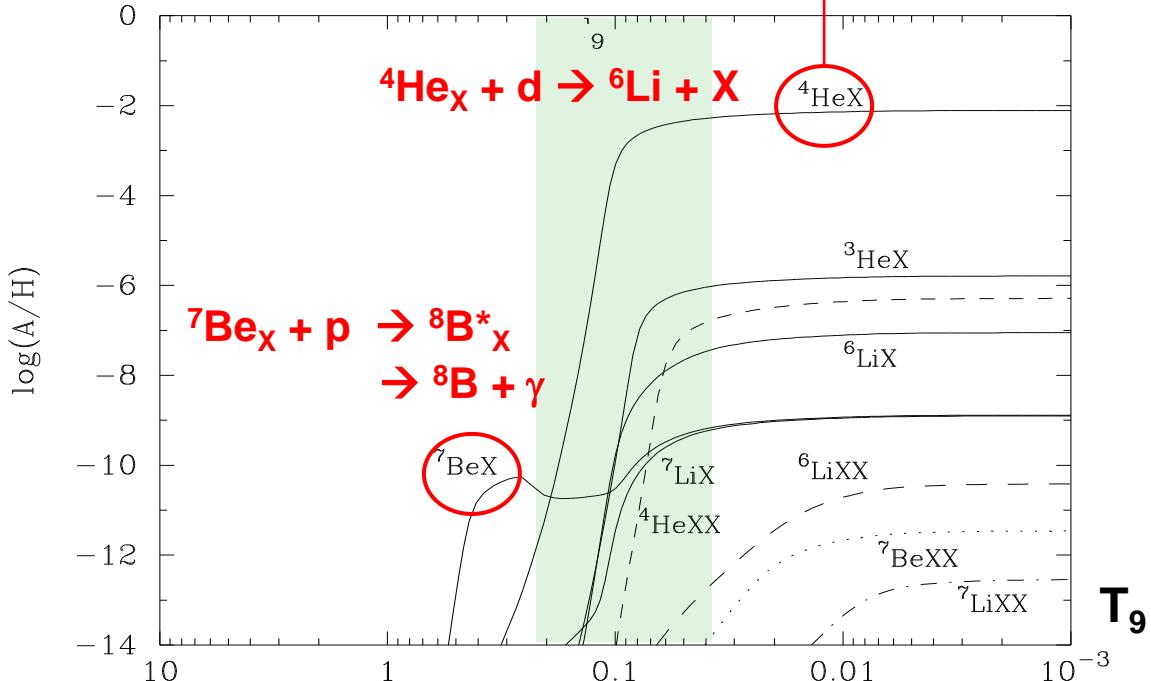
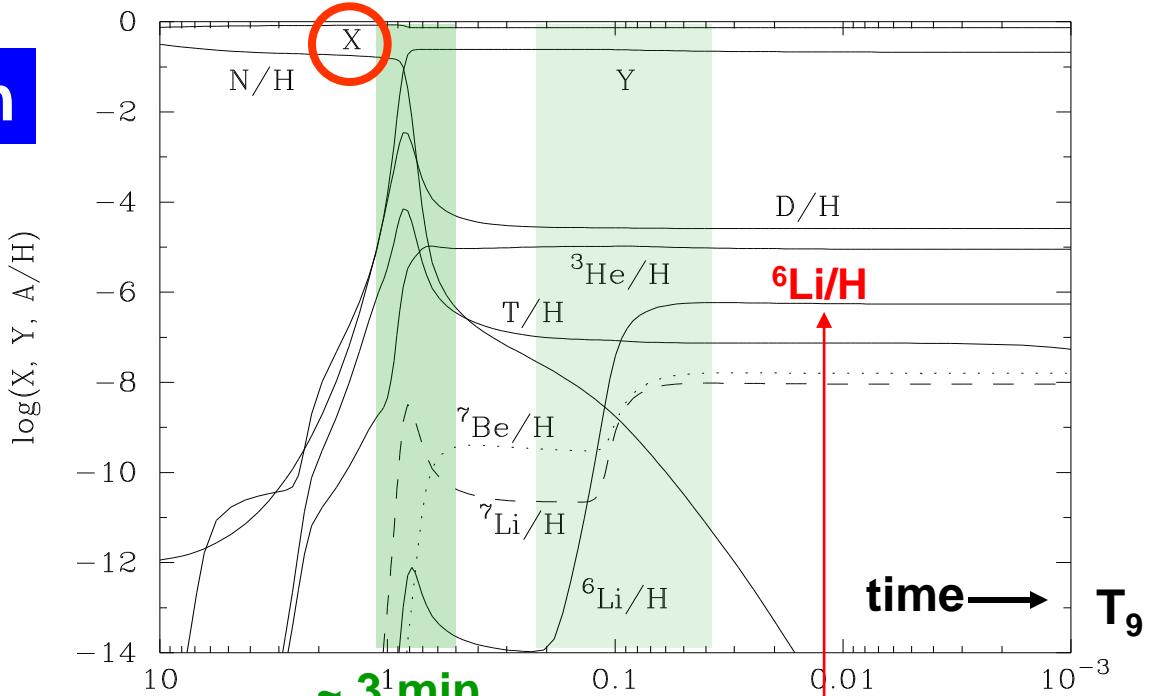
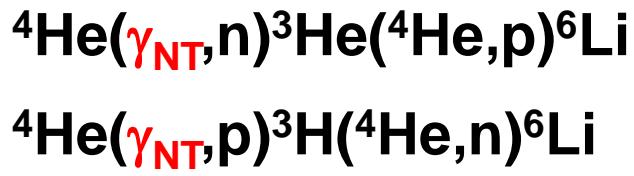
SUSY Leptonic DM (Stau)



Kusakabe, Kajino, Boyd, Yoshida, and Mathews, Phys. Rev. D (2008)

Pospelov (2007)
 Hamaguchi et al. (2007)
 Bird et al. (2007)

Decaying relic DM X



Quest for Nuclear Physics

Stau-catalyzed ${}^6\text{Li}$ production in big-bang nucleosynthesis

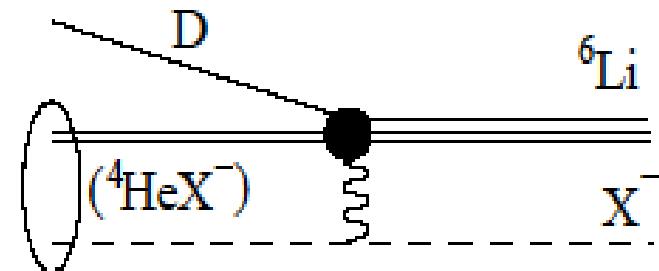
K. Hamaguchi ^{a,*}, T. Hatsuda ^a, M. Kamimura ^b, Y. Kino ^c, T.T. Yanagida ^a

Physics Letters B 650 (2007) 268–274

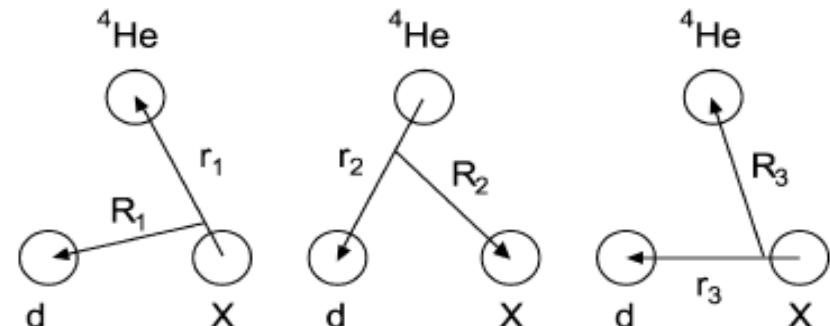
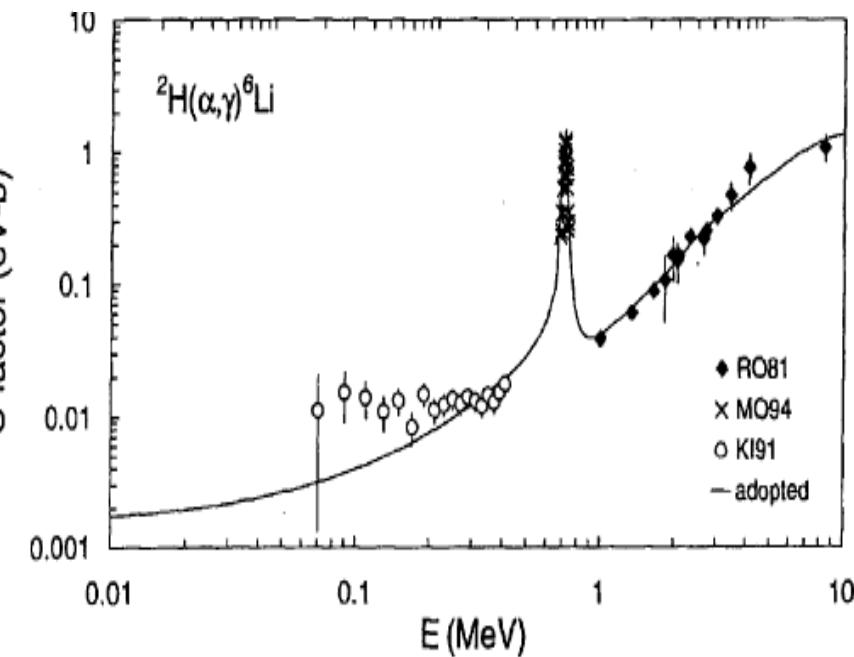
Radiative Fusion Reaction



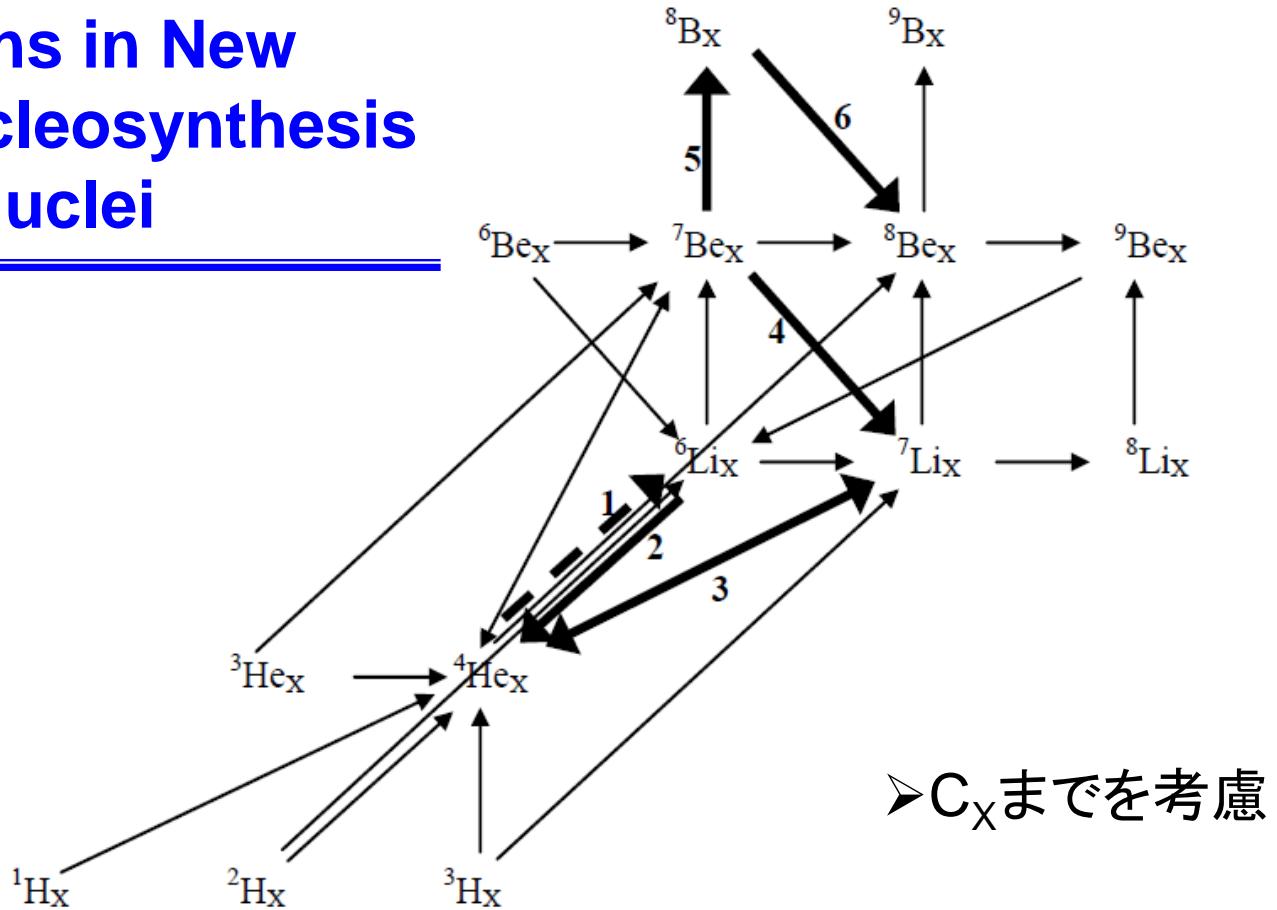
X-Breakup Fusion



Three-Body Clustering Wave Function



Main Reactions in New Big-Bang Nucleosynthesis including X-Nuclei



重要な過程

- 1. ${}^4\text{He}_X(\text{d},\text{X}^-){}^6\text{Li}$
- 2. ${}^6\text{Li}_X(\text{p},{}^3\text{He}\alpha)\text{X}^-$
- 3. ${}^4\text{He}_X(\text{t},\gamma){}^7\text{Li}_X$ & ${}^7\text{Li}_X(\text{p},2\alpha)\text{X}^-$
- 4. ${}^7\text{Be}_X(\text{,X}^0){}^7\text{Li}$
- 5. ${}^7\text{Be}_X(\text{p},\gamma){}^8\text{B}_X$
- 6. ${}^8\text{B}_X(\text{,e}^+\nu_{\text{e}}){}^8\text{Be}_X$

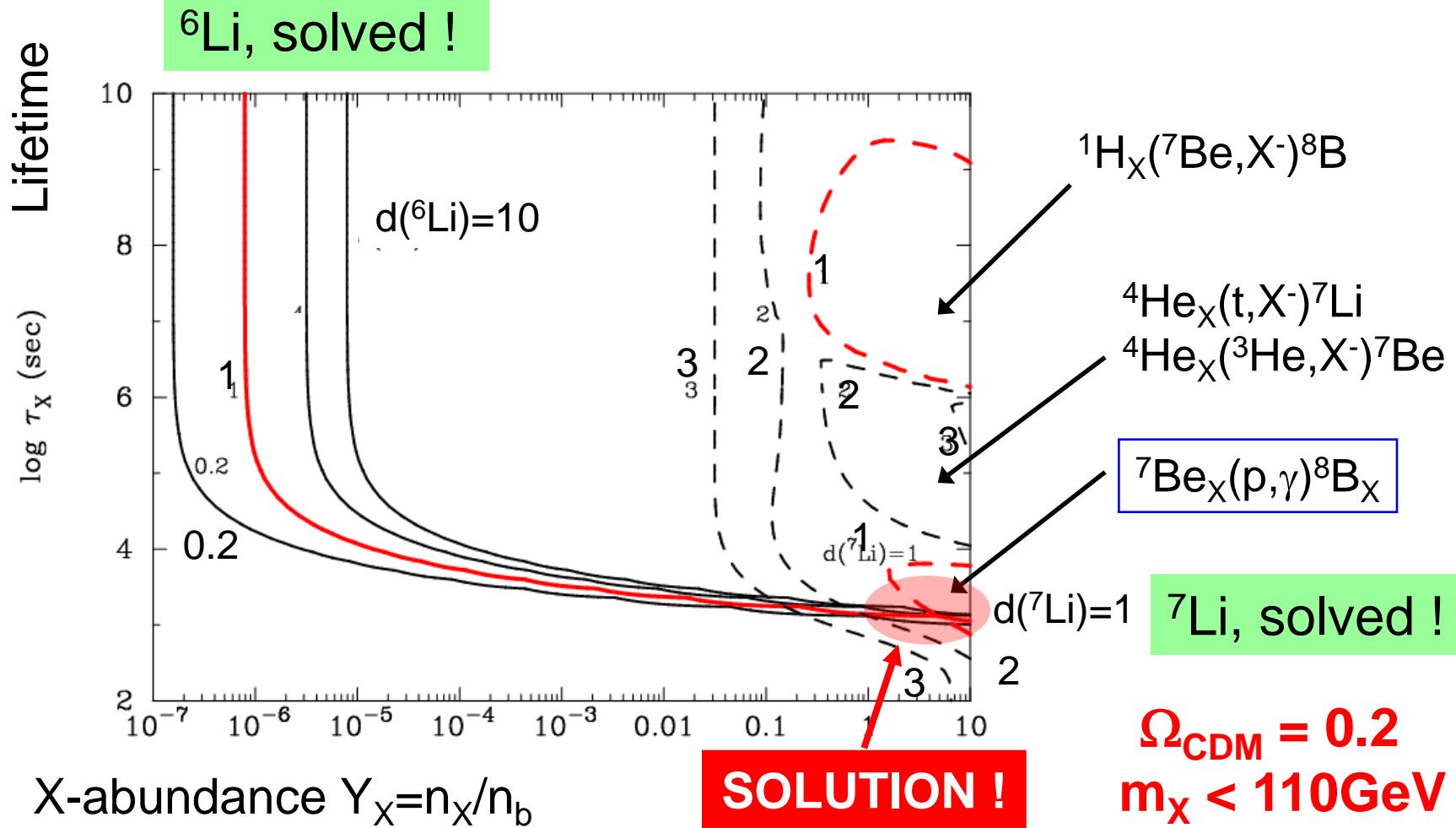
- ✓ X-再結合: 16
- ✓ X核反応: 59
(含β崩壊: 2)
- ✓ X-荷電移行: 3
- ✓ X-decay: 11

Cosmological Solution to both $^{6,7}\text{Li}$ problems

Kusakabe, Kajino, Boyd, Yoshida, and Mathews ApJ 680 (2008), 846: PRD 80 (2009), 103501.
80, 103501 (2009)

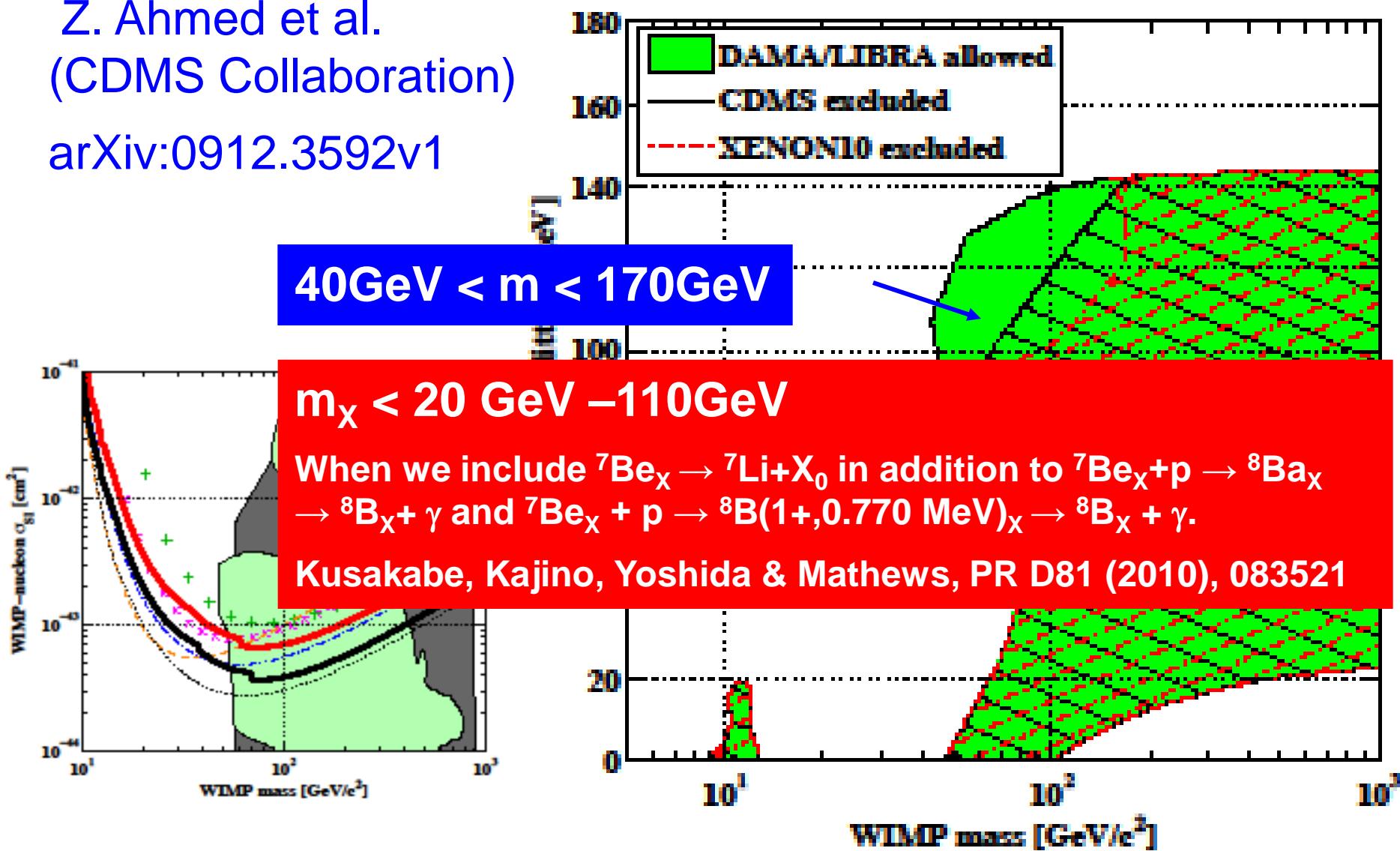
$$d(^A\text{Li}) = ^A\text{Li}^{\text{Calc}} / ^A\text{Li}^{\text{Obs}}$$

$$\eta = 6.1 \times 10^{-10}$$



Results from the Final Exposure of the CDMS II Experiment

Z. Ahmed et al.
(CDMS Collaboration)
arXiv:0912.3592v1



How to solve HIGH ${}^6\text{Li}$ primordial abundance?

Ellis et al. (1986); Moroi and Kawasaki (1994); Jedamzik PRL 84 (2000) 3248;
Cyburt et al., PRD 67 (2003) 103521; Ellis et al. PLB619 (2005) 30;
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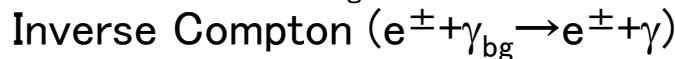
Theoretical of X decay: $X \rightarrow \gamma_{NT}$

Ellis et al. (1986); Moroi and Kawasaki (1994); Jedamzik PRL 84 (2000) 3248; Cyburt et al., PRD 67 (2003) 103521; Ellis et al. PLB619 (2005) 30; Kusakabe, Kajino & Mathews, D74 (2006), 023526.

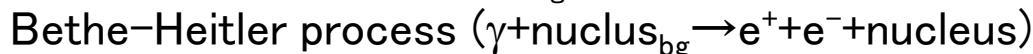
Spectrum of non-thermal γ_{NT}

$$p_\gamma(E_\gamma)$$

Primary γ_{NT} interacts with CBRs



Then it degrades its energy by:



Two Parameters

Life time of X

$$\tau_x$$

Number density * E_γ of X

$$\zeta_x = \frac{n_x^0}{n_\gamma^0} E_{\gamma 0}$$

Reaction process

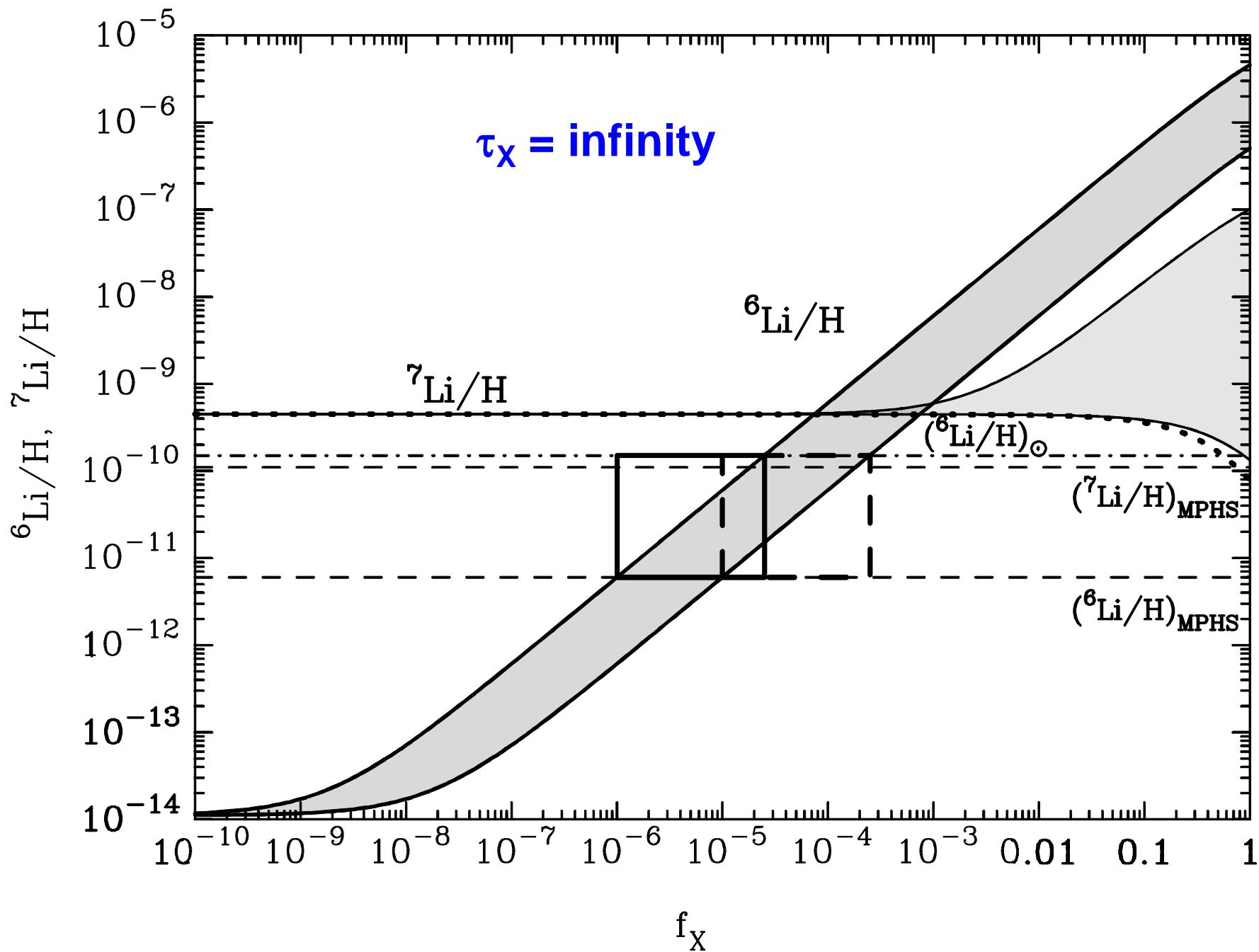
$$\text{Rate equation } \frac{dY_A}{dt} = \sum_P N_A(P) \left(-\frac{Y_A}{N_A(P)!} [A\gamma]_P + \frac{Y_P}{N_P(P)!} [P\gamma]_A \right) + \text{SBBN}$$

$$[A\gamma]_P \equiv \frac{n_\gamma^0 \zeta_x}{\tau_x} \left(\frac{1}{2H_r t} \right)^{3/2} \exp(-t/\tau_x) \int_0^\infty \left(\frac{\tau_x}{E_{\gamma 0} n_x} N_\gamma^{QSE}(E_\gamma) \right) \sigma_{\gamma+A \rightarrow P}(E_\gamma) dE_\gamma$$

Photon # density

$$N_\gamma^{QSE}(E_\gamma) = \frac{n_x p_\gamma(E_\gamma)}{\Gamma_\gamma(E_\gamma) \tau_x}$$

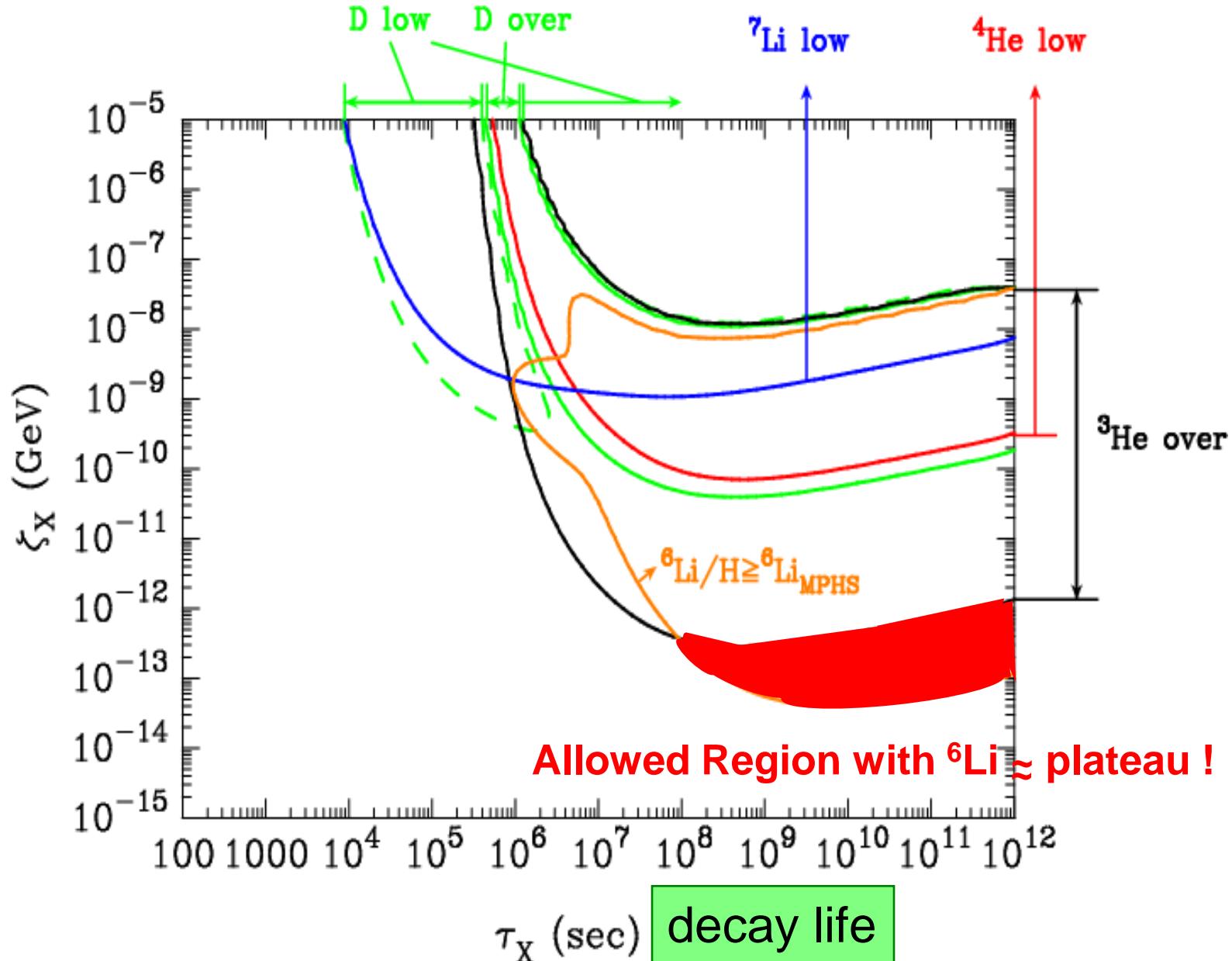
$$H_r = \sqrt{\frac{8\pi G \rho_{rad}^0}{3}}$$



BBN Light Elemental Abundance Constraints on X particle properties

Kusakabe, Kajino & Mathews, Phys. Rev. D74 (2006), 023526.

abundance parameter



Constraint from the CBR energy spectrum

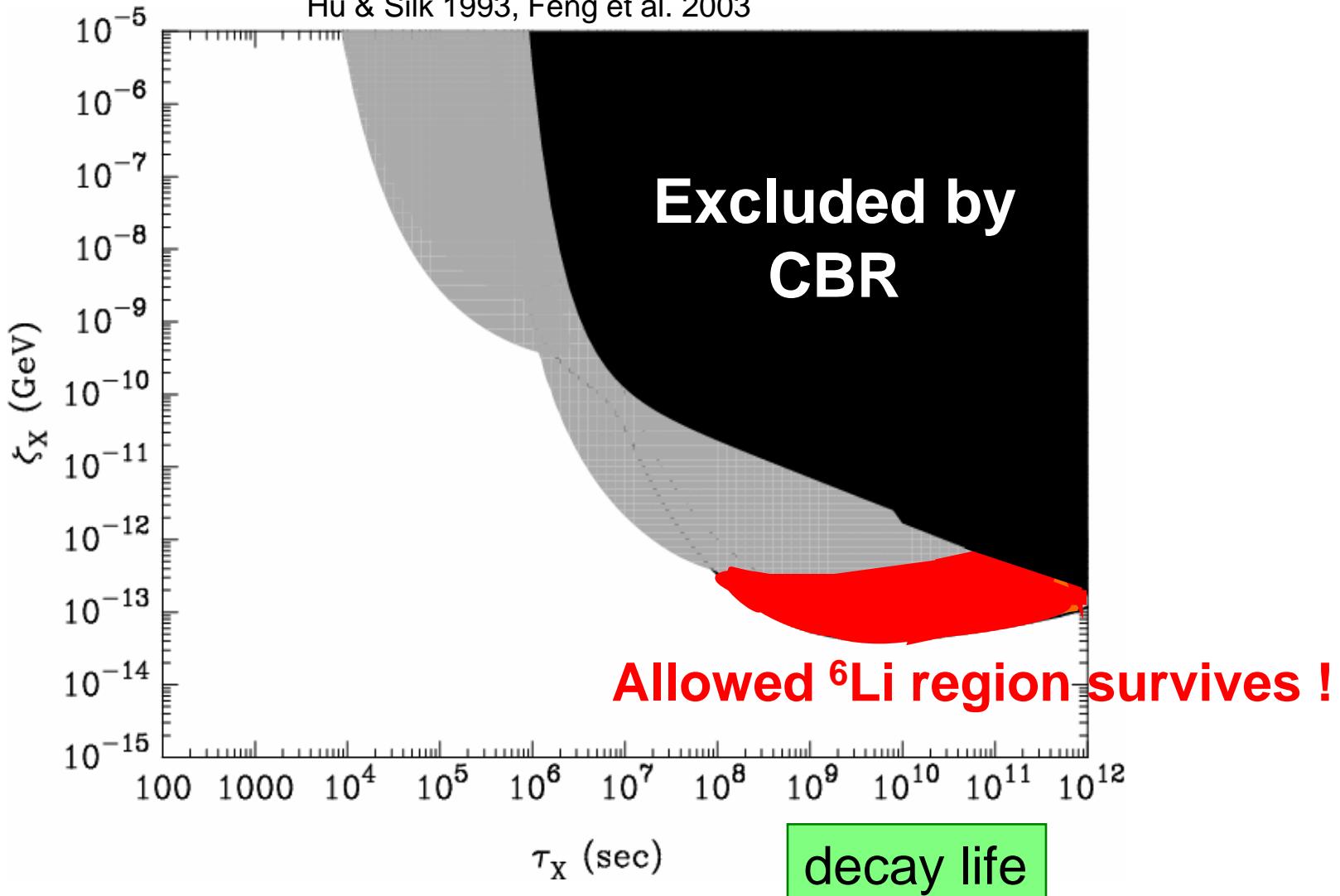
Radiative decay causes CBR distortion from
black-body distribution by $e^\pm \gamma_{BG} \rightarrow e^\pm \gamma$



Another Constraint

Hu & Silk 1993, Feng et al. 2003

abundance parameter



Plateau like HIGH ${}^6\text{Li}$ ABUNDANCE --- primordial ?

Factor 2~4 stellar depletion ?

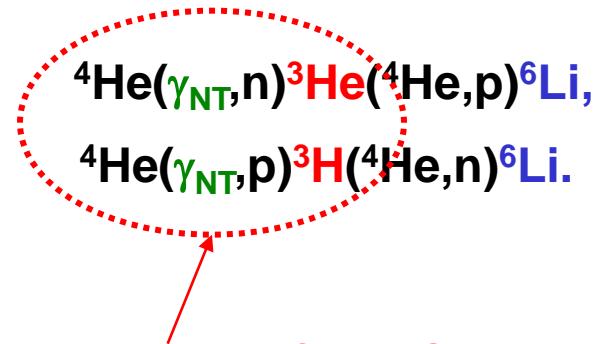
Abundance scatter is too small to accept DEPLETION !

Cosmological radiative decay of
relic CDM X particles
or

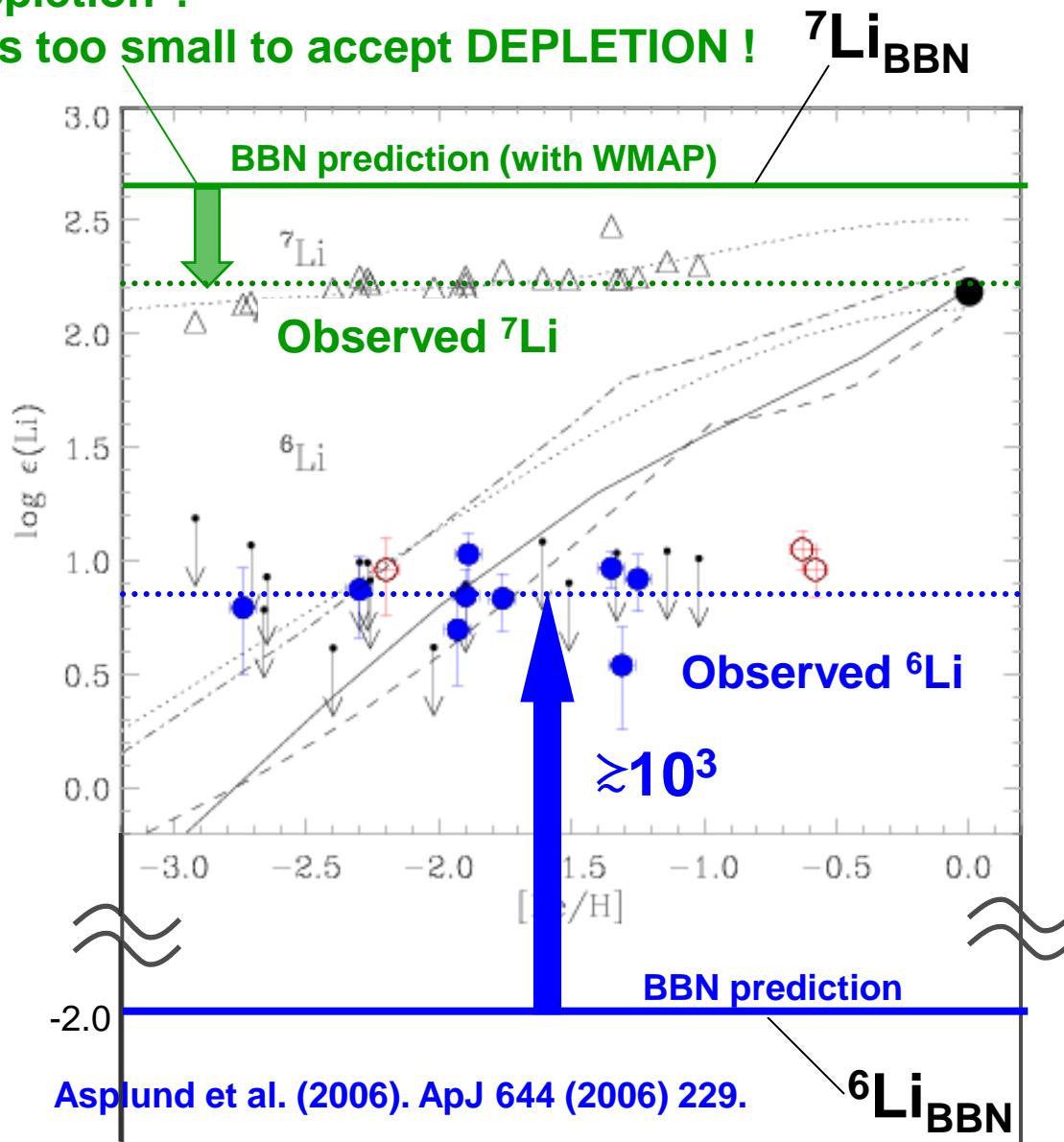
New burst of BBN on leptonic
X-bound nuclei

can resolve
both ${}^6\text{Li}+{}^7\text{Li}$ problems!

Relic X = progenitor of CDM



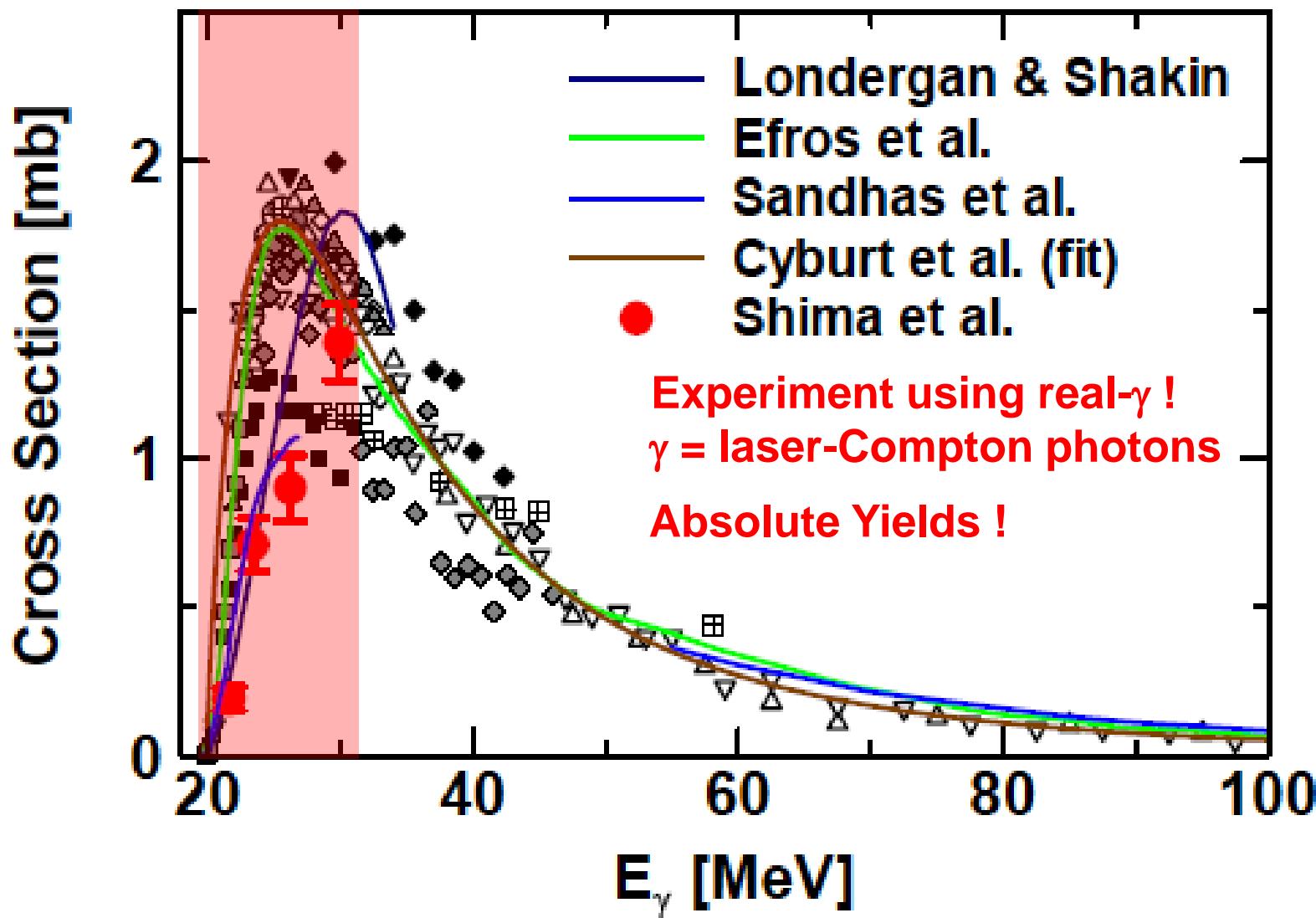
Experimental Cross Sections,
well known ?

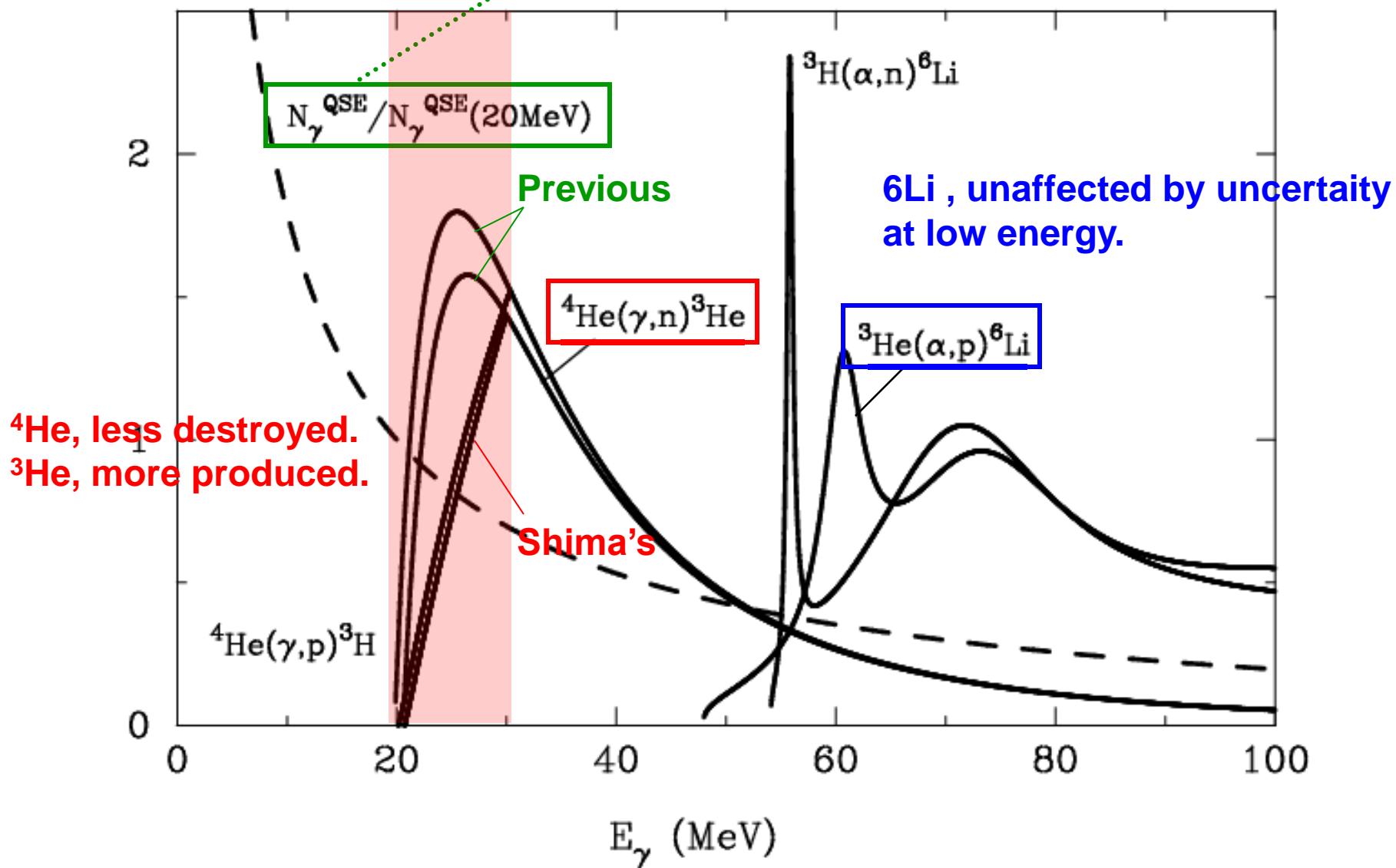


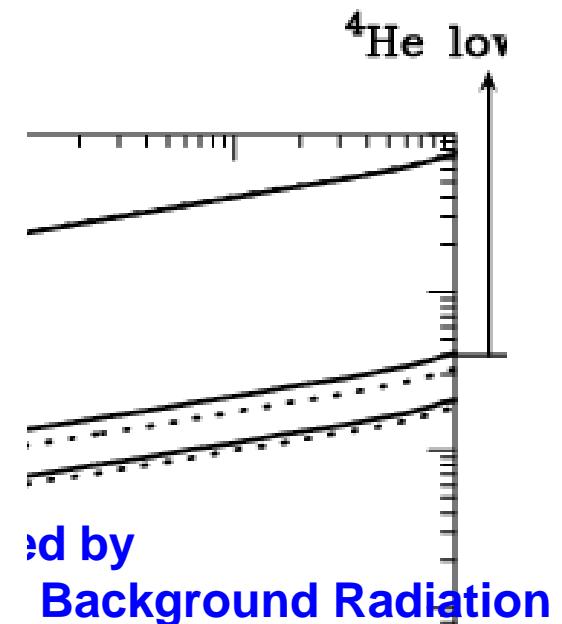
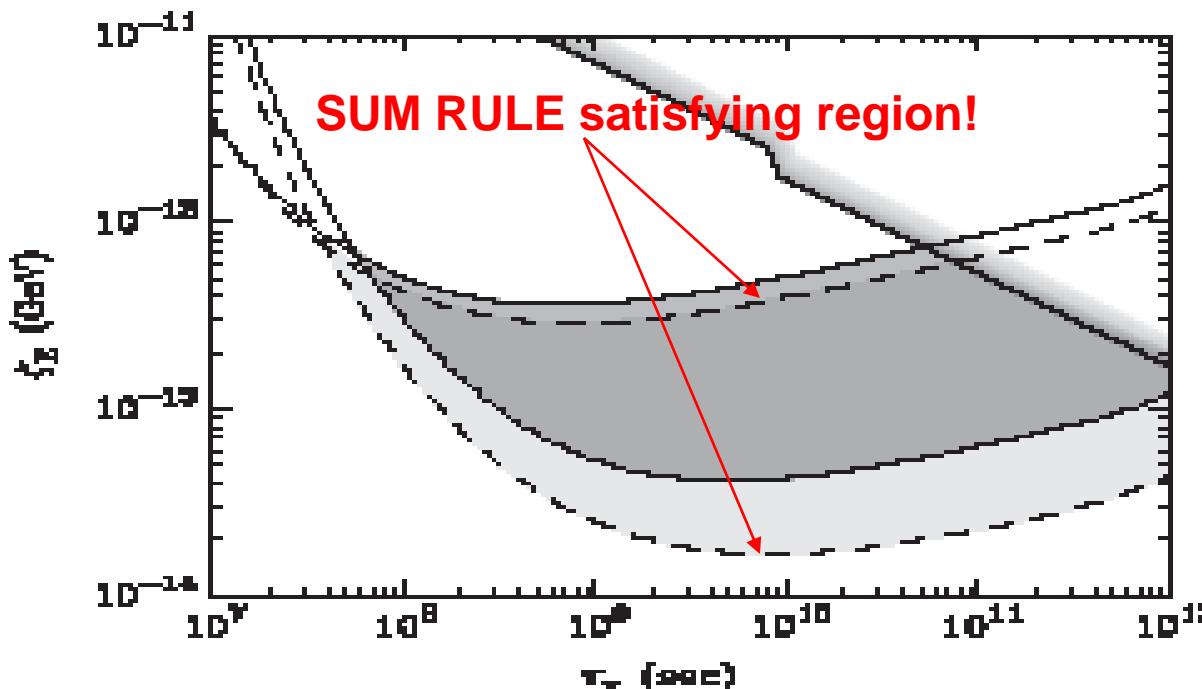
Exp. Data vs. Theor. Calculation

Shima et al. Phys. Rev. C72 (2005) 044004.

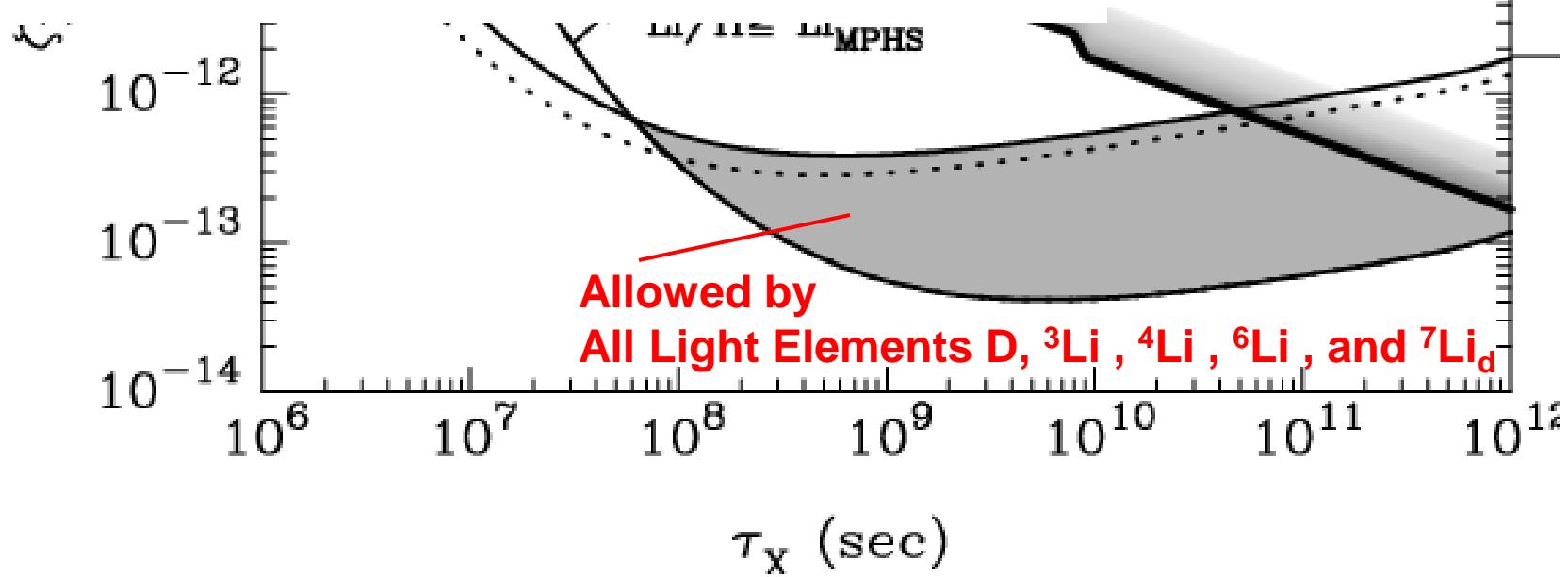
$^4\text{He}(\gamma, \text{p})^3\text{H}$







Allowed by
Background Radiation



原子核の一本演算子に関する二つの和則

Kusakabe, Kajino, Yoshida, Shima, Nagai, and Kii, PRD 79 (2009), 123513.

E1 sum rule

$$\sigma_0 = \int_{E_{th}}^{\infty} \sigma(E_{\gamma}) dE_{\gamma} = \sigma_{TRK} \cdot (1 + \kappa) = 59.74 \times (1 + \kappa)$$
$$\sigma_{TRK} = \frac{2\pi^2 e^2 \hbar}{mc} \cdot \frac{NZ}{A} , \quad \kappa = \langle \Psi_0 | [D_z, [V, D_z]] | \Psi_0 \rangle \times \left(\frac{m}{\hbar^2} \right) \frac{A}{NZ}$$

Thomas-Reihe-Kuhn Meson-exchange current

QM calculation $\kappa = 0.67 \sim 1.14 \Rightarrow \sigma_0 = 100 \sim 128$ [MeV·mb]

Bremsstrahlung Sum rule

$$\sigma_B = \int_{E_{th}}^{\infty} \frac{\sigma(E_{\gamma})}{E_{\gamma}} dE_{\gamma} = 4\pi^2 \frac{e^2}{\hbar c} \times \langle \Psi_0 | D_z D_z | \Psi_0 \rangle$$
$$= \frac{4\pi^2}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{NZ}{A-1} \cdot \left(\langle r_p^2 \rangle - \langle r_{\alpha}^2 \rangle \right) = 2.62 \pm 0.02 \text{ [mb]}$$

Proton's ms charge radius $\langle r_p^2 \rangle^{1/2} = 0.870 \pm 0.008$ [fm]

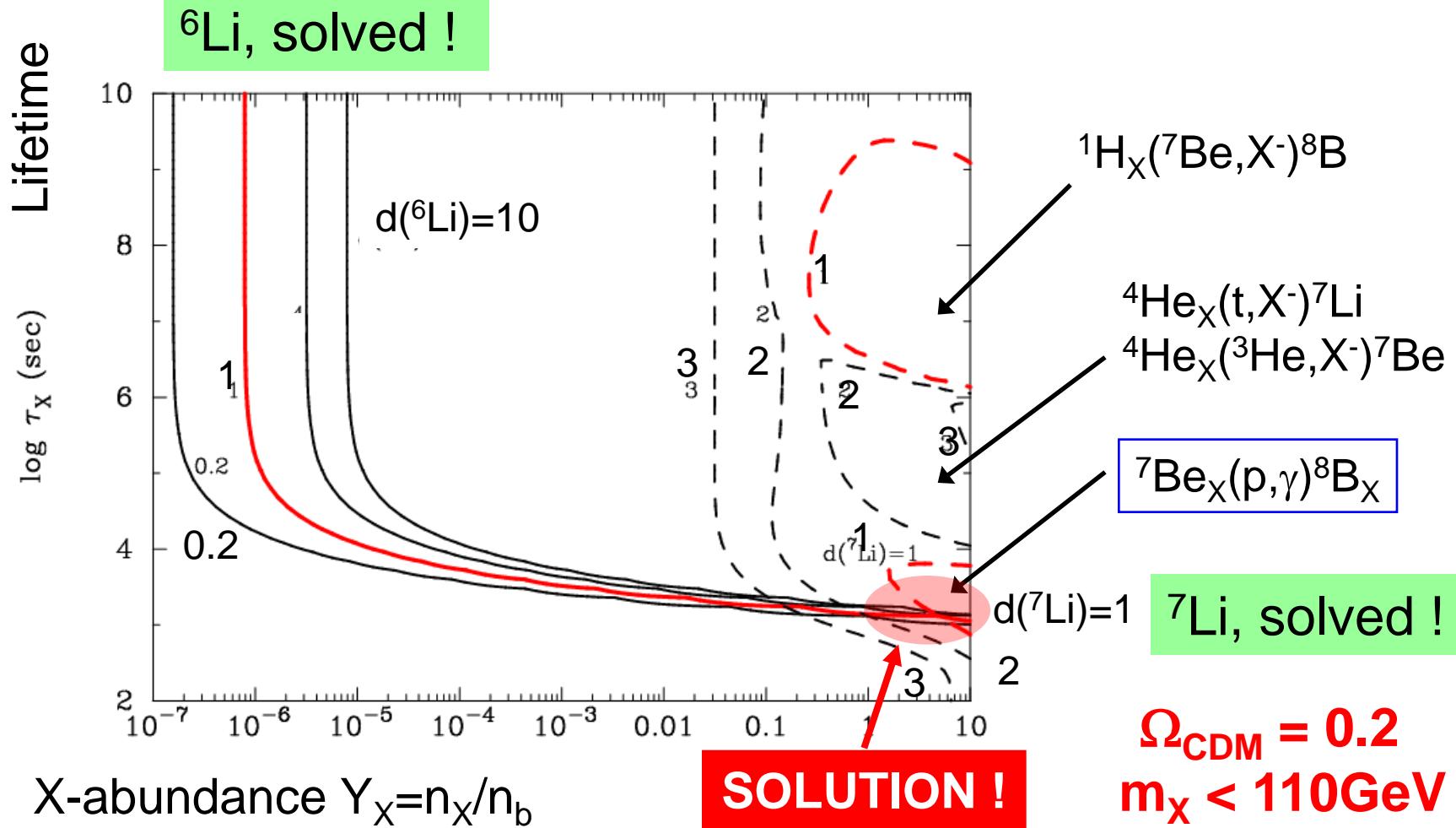
${}^4\text{He}$'s ms charge radius $\langle r_{\alpha}^2 \rangle^{1/2} = 1.673 \pm 0.001$ [fm]

Cosmological Solution to both $^{6,7}\text{Li}$ problems

Kusakabe, Kajino, Boyd, Yoshida, and Mathews ApJ 680 (2008), 846: PRD 80 (2009), 103501.
80, 103501 (2009)

$$d(^A\text{Li}) = ^A\text{Li}^{\text{Calc}} / ^A\text{Li}^{\text{Obs}}$$

$$\eta = 6.1 \times 10^{-10}$$



Particle Physics Experiment tests Cosmological and Astronomical Prediction !



Conclusion

Big-Bang Nucleosynthesis provides “critical test” for the particle and nuclear models of weak interactions (neutron decay in quark level) and even cosmological models (through CBR constraints on DM particles or th end of Inflation).

Relic SUSY particle - CDM model, if they are bound in normal nuclei or radiative decay to non-thermal photons, can solve both 6Li and 7Li problems in astronomy concerning primordial Big-Bang nucleosynthesis.