格子QCDによるクォーク多体系の物理

Multi-Quark Physics with Lattice QCD

H. Suganuma (Kyoto Univ.)
in collaboration with T.T.Takahashi (YITP), F.Okiharu (Niigata),
K.Tsumura (Fuji-film), N.Ishii (Tsukuba), A.Yamamoto (Kyoto)

Nowadays, lattice QCD is a powerful tool to analyze nonperturbative QCD quantitatively, as the first-principle calculation of the strong interaction.

Menu

1. Introduction
2. Tetra-Quark Candidates in Light Quark Sector
3. A brief review of Lattice QCD for hadron mass calculations
4. Difficulty of Multi-Quark calculations in Lattice QCD
5. Lattice QCD Studies for Light Tetra-Quarks
6. Multi-Quark Potential in Lattice QCD
   ~ other possible approach combined with quark-model calculation

JPS Spring Meeting, March 30 2009, Rikkyo

Multi-Quark Physics

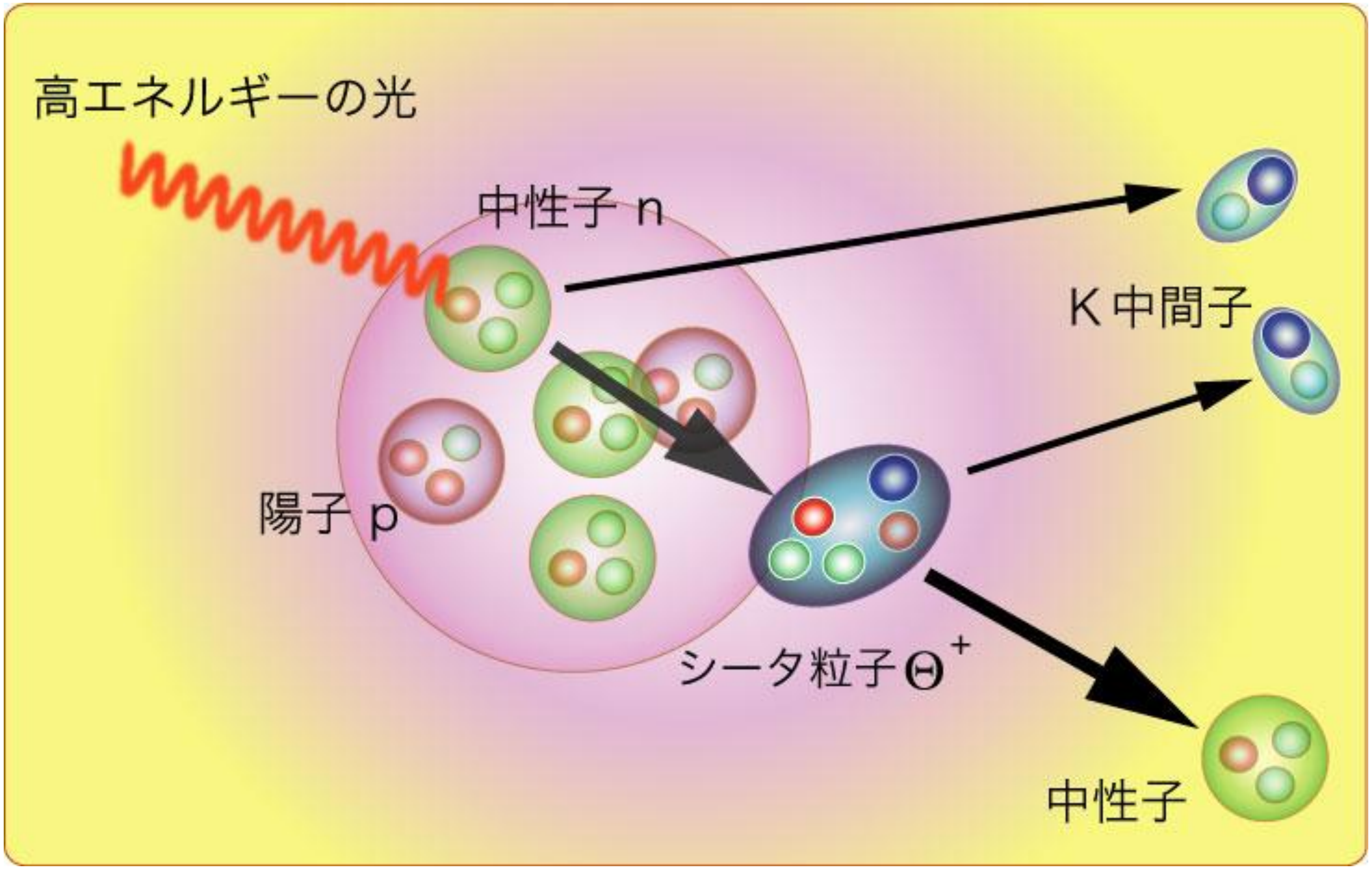
- After the experimental report of penta-quark candidate $\Theta^+(1530)$ at SPring-8, lots of theoretical analyses for the exotic hadrons have been done or revisited.

- Of course, Exotic hadrons (multi-quarks, hybrids, glueballs) are very interesting objects beyond the simple quark model.

- In these years, several charmed tetra-quark candidates such as X(3872) have been experimentally discovered, and Tetra-Quark has been also investigated as an interesting object in quark-hadron physics.

- Very recently, the discovery of a "*charged charmonium*" $Z^+(4430)$ $(c\bar{c}u\bar{d})$ with exotic quantum numbers is reported at KEK-Belle experiment.

Multi-Quark Search and Multi-Quark Physics



The experimental report on discovery of $\Theta^+(1530)$ （LEPS, DIANA, CLAS, SAPHIR ）created not only a new particle but also a New Frontier of “Multi-Quark Physics”.

Multi-Quark Physics

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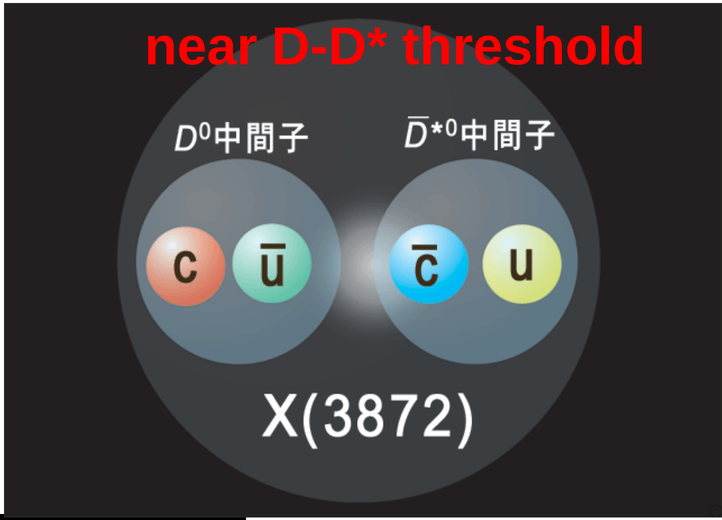
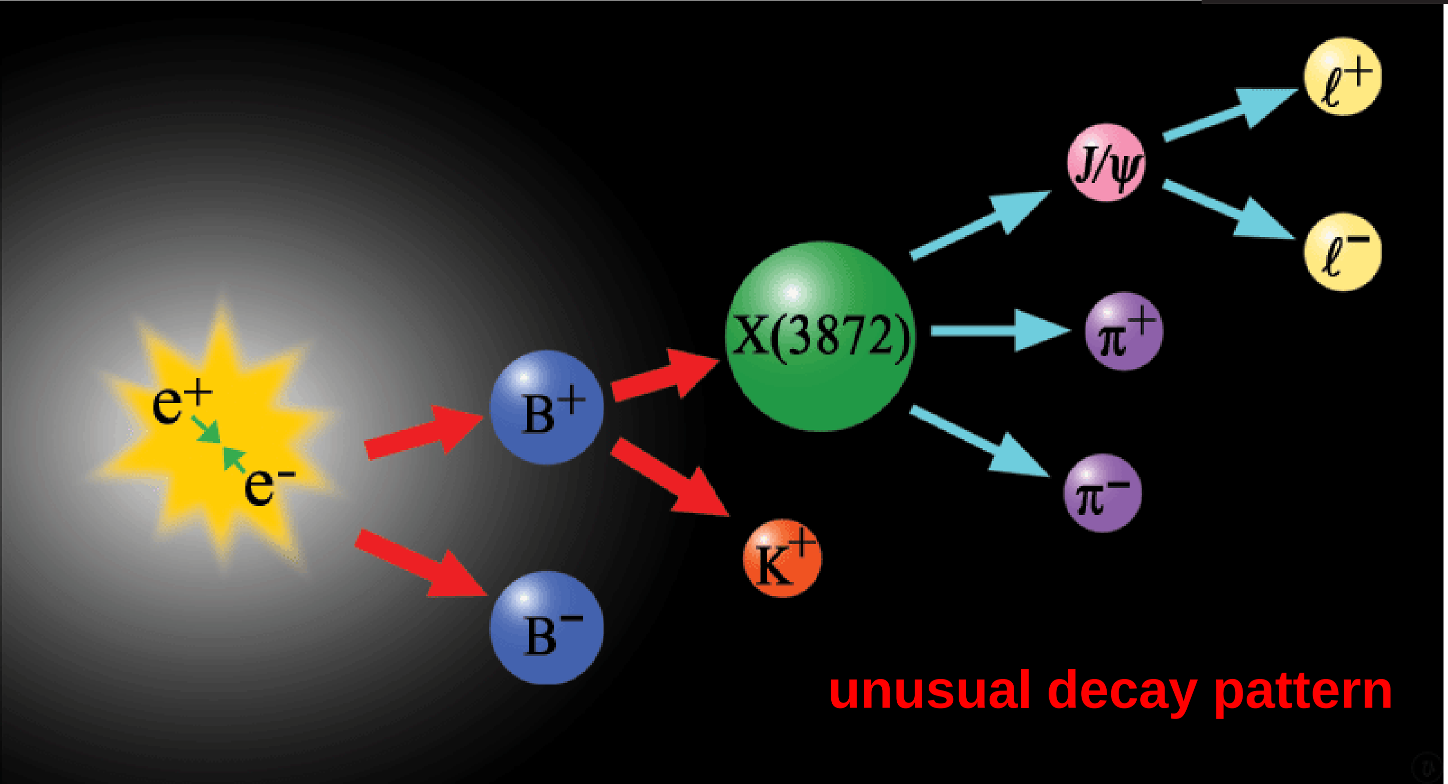
Exotic Hadrons：New-type of quantum many-body system

Experimental discovery of
Exotic Charmed Hadron candidates
have been done at KEK（Belle）, SLAC（BaBar）:

Tetra-Quark or Hybrid candidates
X(3872), Y(3940), $D_{s0}^{+}(2317)$ etc.

→ This finding gives a Strong Impact to
QCD and quark-hadron physics

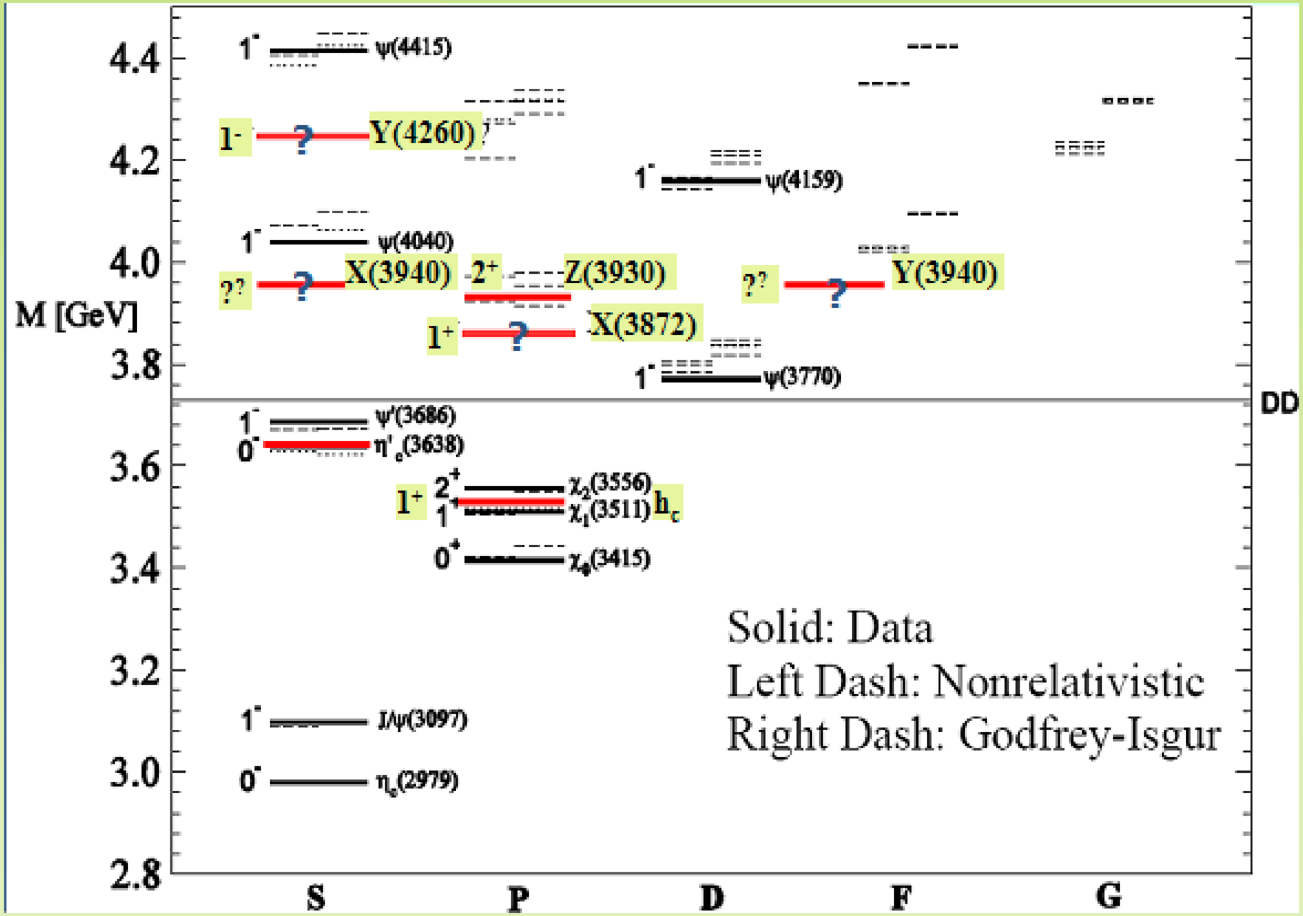




Figs. from KEK web-site

From Prof. A. Bonder's Talks

Charmonium spectroscopy



YKIS06, Kyoto November 10, 2006 A.Bondar

So many new-type hadrons which cannot be explained with the simple quark model have been observed.

→ We need theoretical Analysis of Multi-Quarks or Hybrids based on QCD

Multi-Quark Physics

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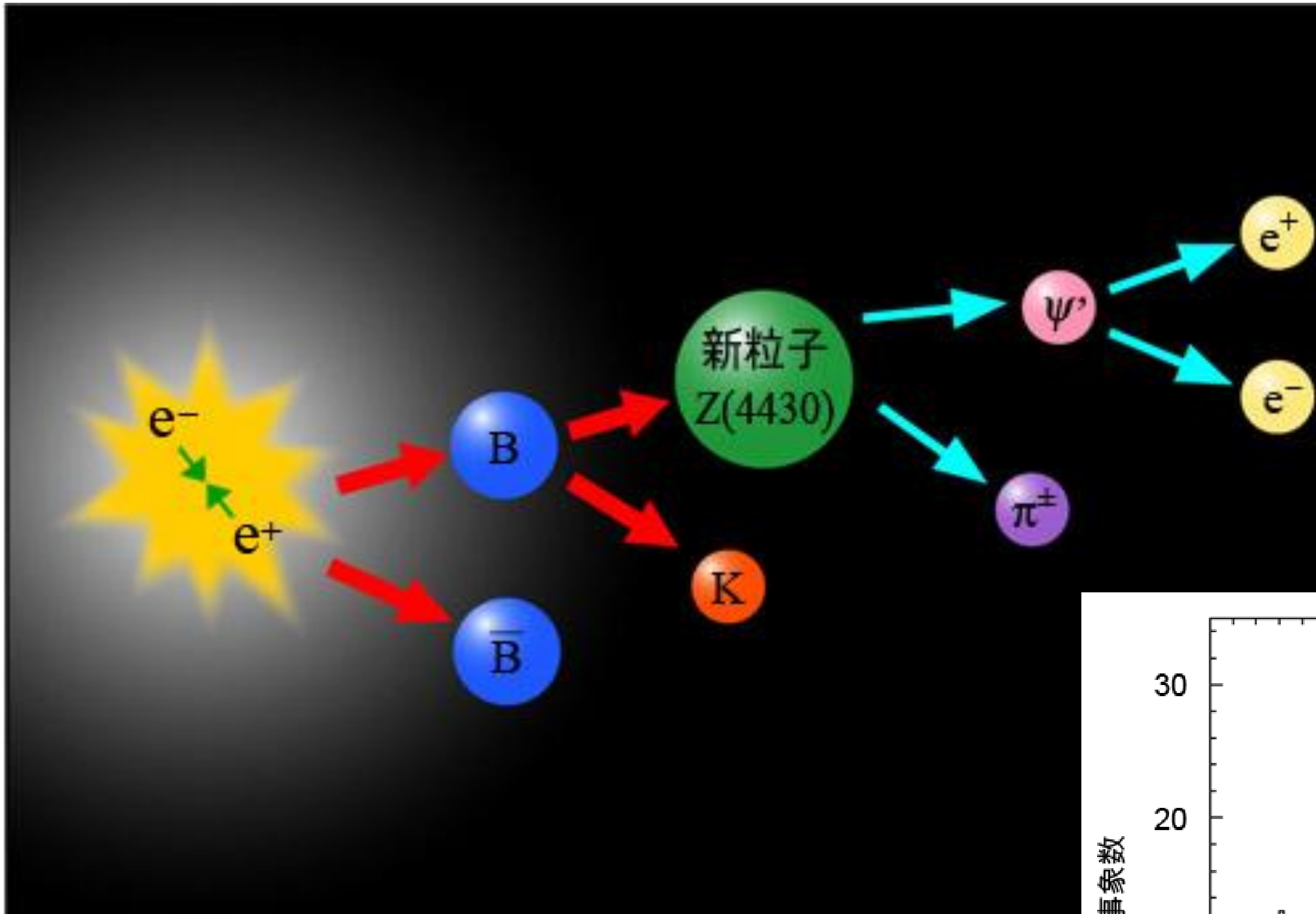
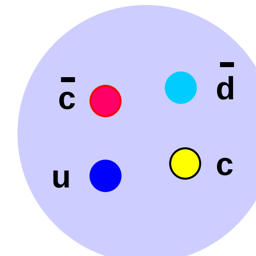
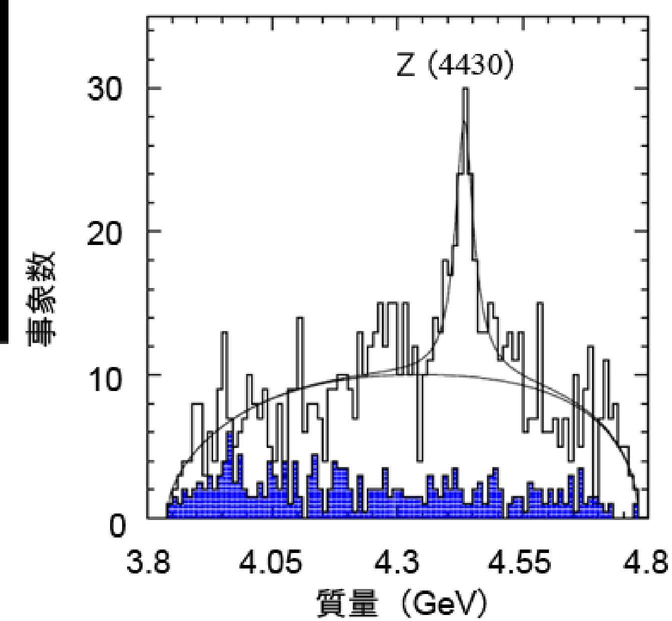
- In these years, several charmed tetra-quark candidates such as X(3872) have been experimentally discovered, and Tetra-Quark has been also investigated as an interesting object in quark-hadron physics.

- Very recently, the discovery of a "*charged charmonium*" $Z^+(4430)$ $(c\bar{c}u\bar{d})$ with exotic quantum numbers is reported at KEK-Belle experiment.

Tetra-Quark Z(4430) from KEK press release







In particular, the *charged charmonium* $Z^{+}(4430)$ is important because it is a *manifest Tetra-Quark hadron* composed by $c\bar{c}u\bar{d}$.

Multi-Quark Hadrons with Charm or Strange Quark

The discoveries of multi-quark candidates, $\Theta^+(1530)$, $X(3872)$, $Z^+(4430)$ $(c\bar{c}u\bar{d})$, have a large impact to the theoretical side.

Experiments: Belle, BaBar, BES, Cleo ....

Possible interpretations: tetraquarks [Maiani, Polosa, ...]

$$Z(4430) \to J/\psi\ \pi^+ : [cu]^{S=0}[\bar{c}\bar{d}]^{S=1} + [cu]^{S=1}[\bar{c}\bar{d}]^{S=0}, \quad 2S$$

$$Y(4260) : [cs]^{S=0}[\bar{c}\bar{s}]^{S=0} \quad P$$

$$X(3872) : [cd]^{S=0}[\bar{c}\bar{d}]^{S=1} + [cd]^{S=1}[\bar{c}\bar{d}]^{S=0} \quad 1S$$

$$X(3875) : [cu]^{S=0}[\bar{c}\bar{u}]^{S=1} + [cu]^{S=1}[\bar{c}\bar{u}]^{S=0} \quad 1S$$

This "new" subject, Tetra-Quark Physics, also relates to the old famous problem called as "scalar meson puzzle" in the light-quark sector.

Tetra-Quark Candidates in Light Quark Sector

Theoretical Conjecture for Light Tetra-Quark Systems

Simple quark model

q and $\bar{q}$ have opposite parity

↓

Parity of q $\bar{q}$ meson : $P=(-1)^{L+1}$

Pseudo-scalar meson

$J^P=0^-$

q $\bar{q}$

L=0

light

Scalar meson

$J^P=0^+$

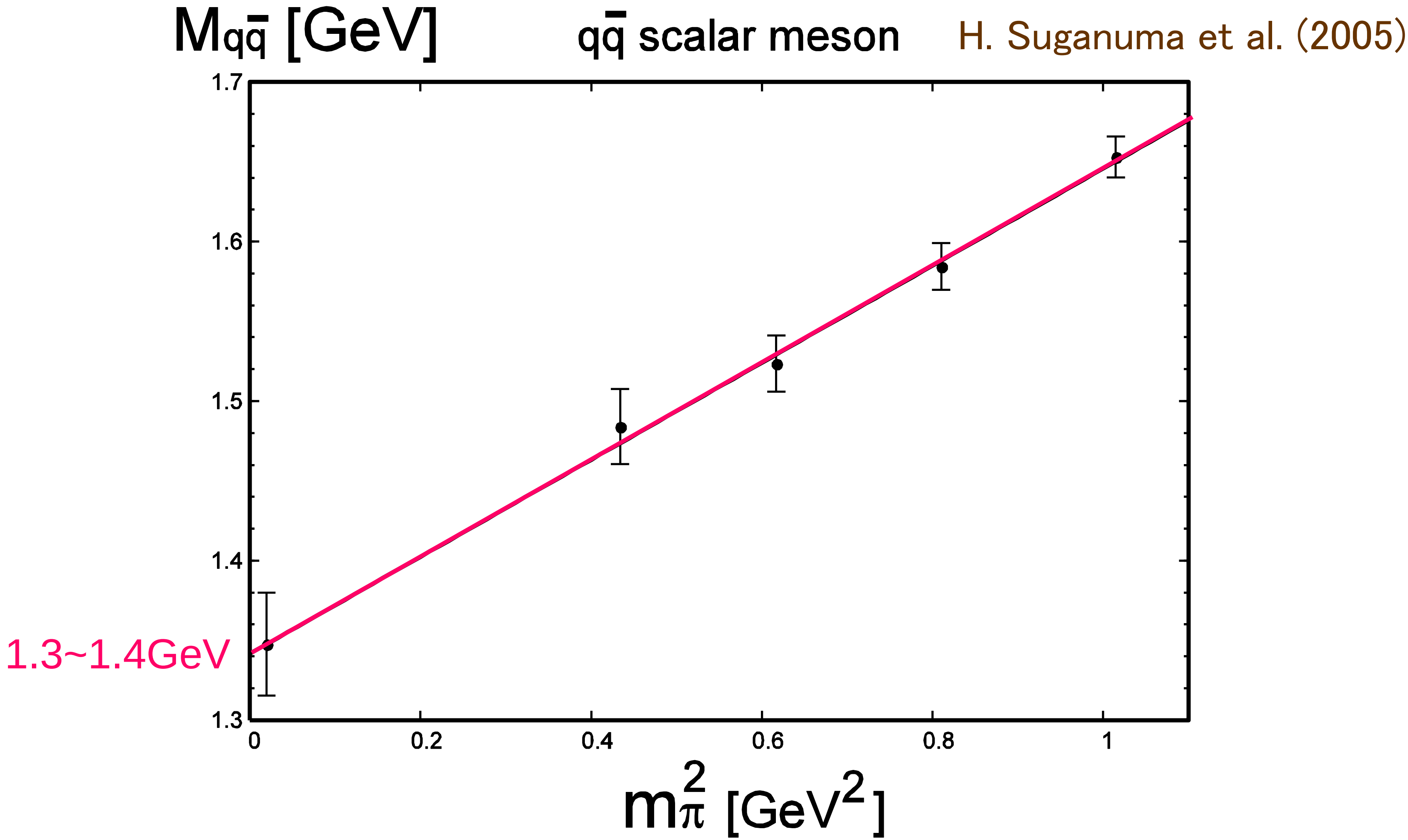
q $\bar{q}$

L=1

should be heavy

$q\bar{q}$ scalar meson is P-wave (L=1) , and hence its mass should be large.
~ Particle Data Group identifies $q\bar{q}$ scalar meson as
$f_0(1370)(I=0)$, $a_0(1450)$ $(I=1)$.

$q\bar{q}$ scalar meson in Lattice QCD



$q\bar{q}$ scalar meson mass is about 1.3~1.4GeV

→ $f_0(1370)(I=0)$, $a_0(1450)(I=1)$

"Scalar Meson Puzzle" and
Tetra-quark Candidates in the Light-Quark Sector

Also in the light-quark sector, there are tetra-quark candidates.

- There are five 0^{++} isoscalar mesons below 2GeV:

  $f_0(400\text{-}1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$.

- Among them, $f_0(1500)$ and $f_0(1710)$ are expected to be the lowest scalar glueball or an $s\bar{s}$ scalar meson.

- $f_0(1370)$ is considered as the lowest $q\bar{q}$ scalar meson in the quark model. For, in the quark model, the lowest $q\bar{q}$ scalar meson is $^3P_0$, and therefore it turns to be rather heavy.

- So, what are the two light scalar mesons, $f_0(400\text{-}1200)$ and $f_0(980)$? This is the "scalar meson puzzle", which is unsolved even at present.

- As a possible answer, Jaffe proposed tetraquark ($qq\bar{q}\bar{q}$) assignment for low-lying scalar mesons such as $\sigma(600)$, $f_0(980)$ (I=0), $a_0(980)$(I=1) in 1977.

Theoretical Conjecture for Light Tetra-Quark Systems

Simple quark model

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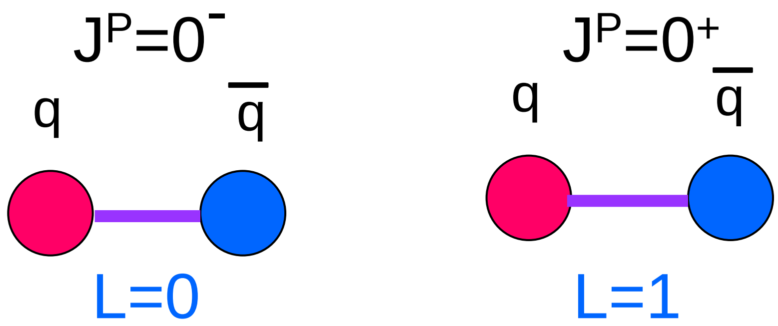
↓

Parity of q $\bar{q}$ meson : $P=(-1)^{L+1}$

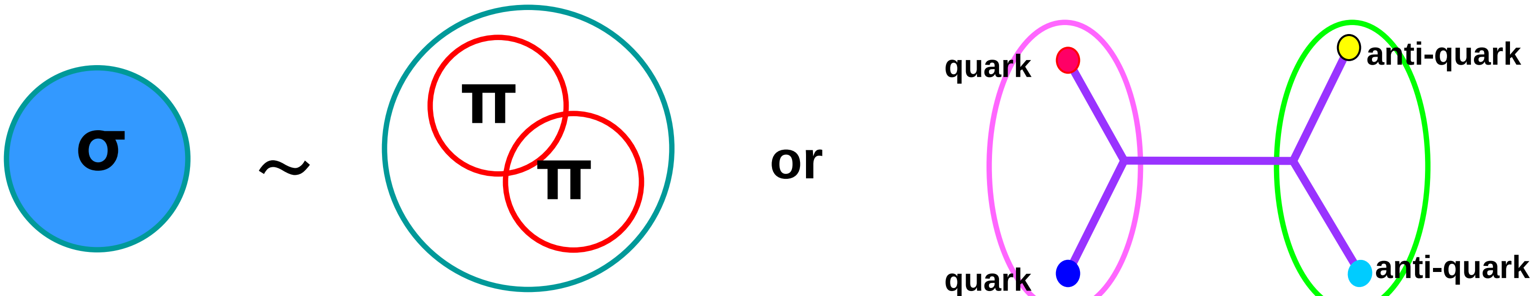
Pseudo-scalar meson | **Scalar meson**



light | **should be heavy**

$q\bar{q}$ scalar meson is P-wave (L=1) , and hence its mass should be large.
~ Particle Data Group identifies $q\bar{q}$ scalar meson as $f_0(1370)$(I=0), $a_0(1450)$ (I=1).

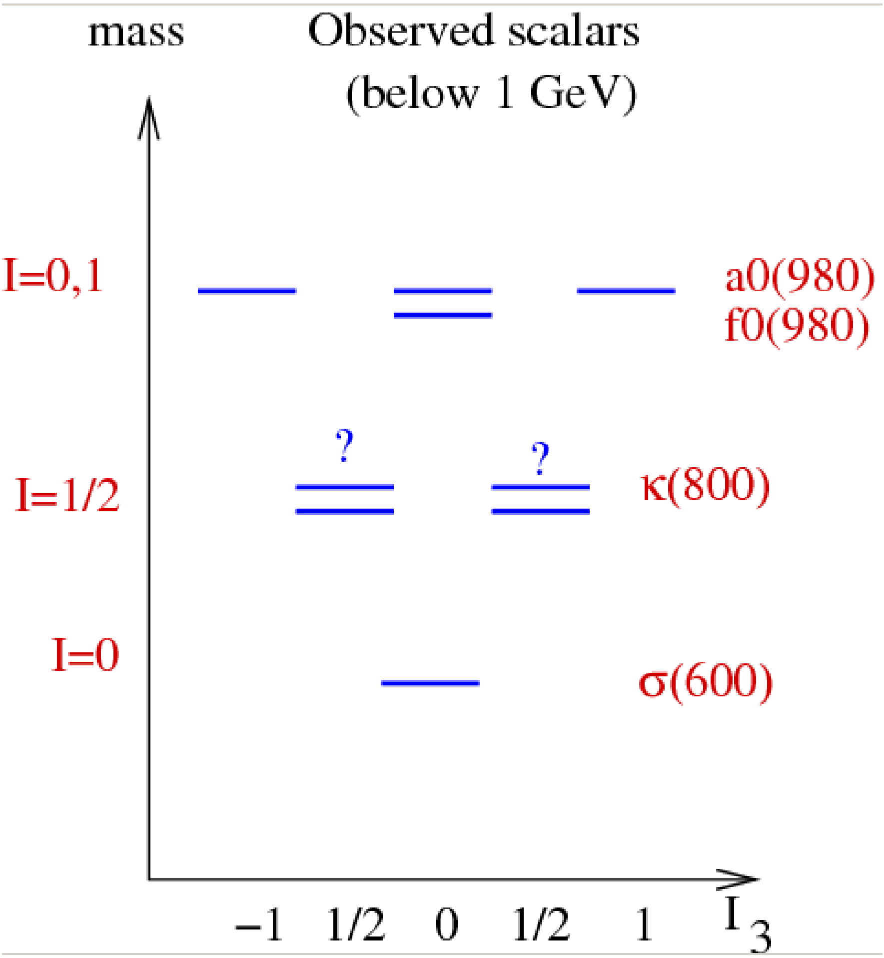
▪ $f_0(980)$, $a_0(980)$, σ(600) : light scalar mesons ~ tetra-quark candidates



hadronic molecule

tetra-quark state

Scalar nonet hypothesis in flavor SU(3) (Jaffe)



In flavor SU(3) sector, tetra-quark hadrons forms scalar nonet for scalar mesons.

Actually, the nonet candidate scalar mesons are observed.

… and, such mass ordering can be also explained by the tetra-quark picture for scalar mesons.

Not only Charmed Tetra-Quarks but also Scalar nonet are desired to be examined in lattice QCD

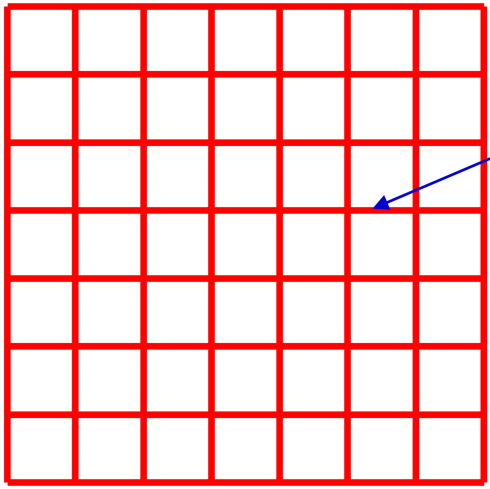
A brief review of Lattice QCD for hadron mass calculations

Lattice QCD

~First Principle Calculation of Strong Interaction~

Based on the Directly numerical calculation of
Generating Functional of QCD
on a discretized space-time in Euclidean metric

$$Z_{QCD}=\int DqD\bar{q}DA\; e^{-S_{QCD}[q,\bar{q},A]}$$



$$A_\mu^a(x) \rightarrow A_\mu^a(x_n)$$

μ=0,1,2,3

Color index
a=1,2,…,8

Lattice site
(huge numbers)

For example, in the case of 16^4 lattice, degrees of freedom of Gluon field $A_\mu^a(x)$ are 16^4 x 4 x 8 = 2,097,152. The gluonic part in lattice QCD is expressed as about 2 milion multiple integral.

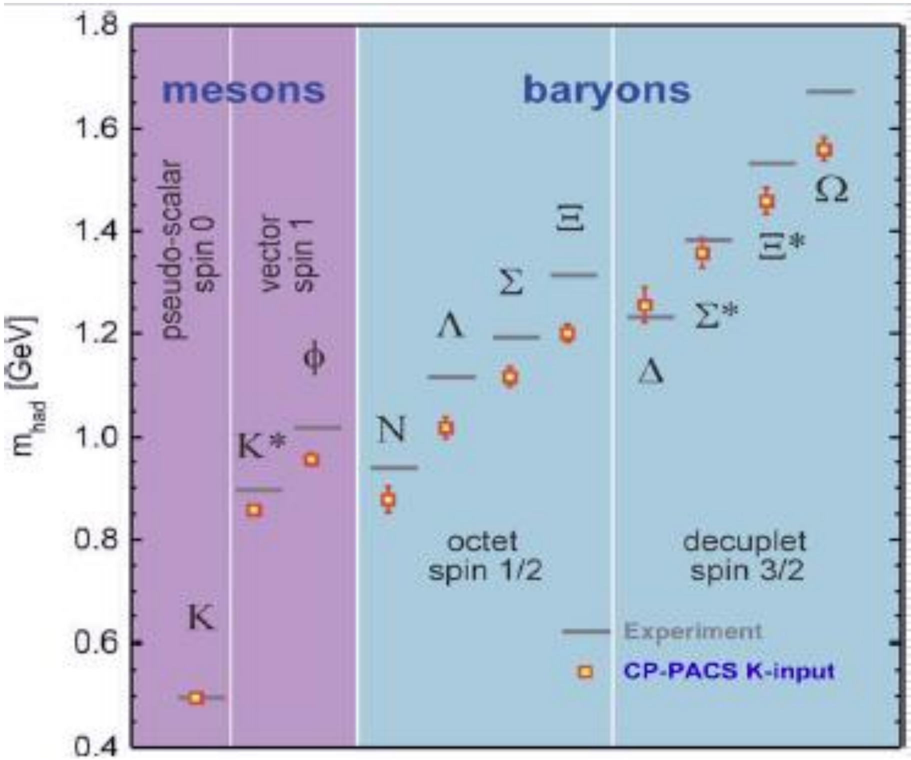
NEC-SX8 @ RCNP

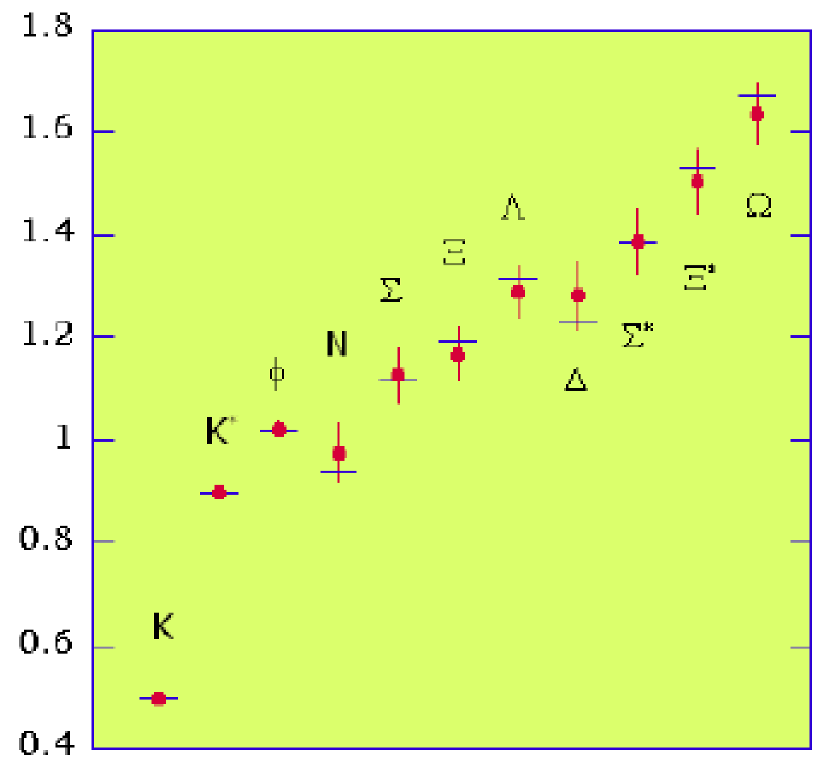
PACS-CS @ Tsukuba

BlueGene/L @ KEK

These SuperComputers eneable us to calculate several milion multiple integral

Hadron Mass Calculation in Lattice QCD





CP-PACS

Quenched QCD

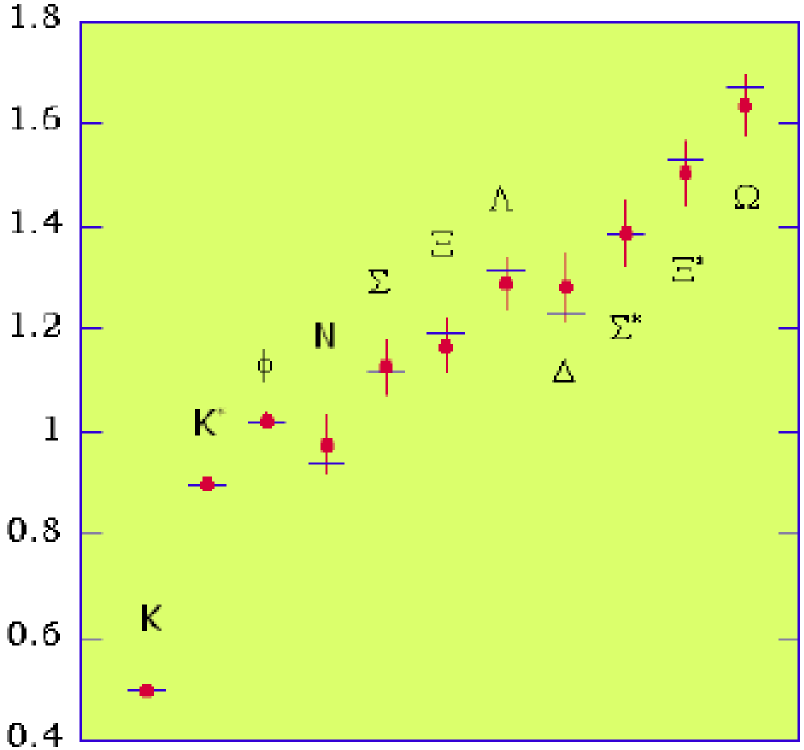
(no dynamical quark-pair creation/annihilation)

Full QCD

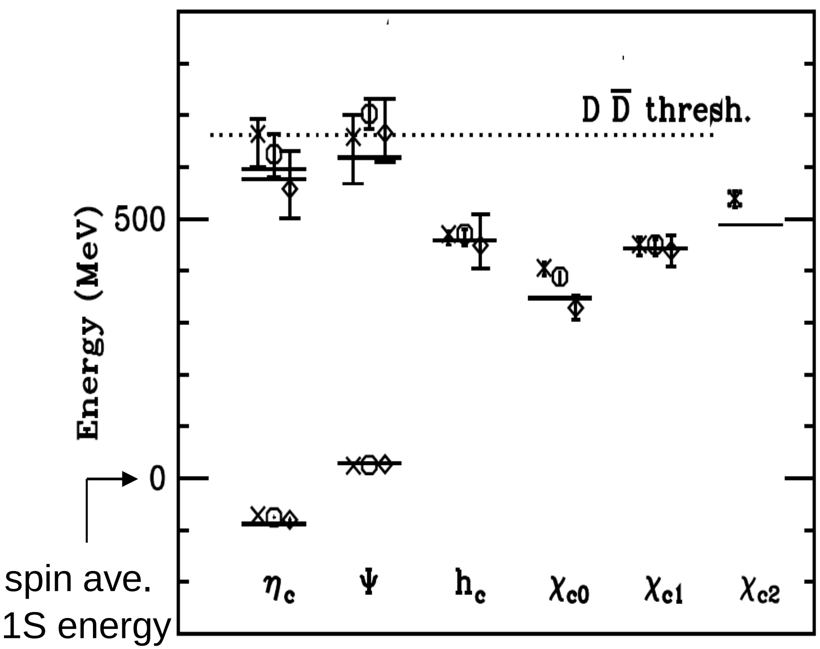
For each quantum number (spin, isospin, strangeness), the lowest hadron mass is well reproduced in Lattice QCD

Hadron Mass Calculation in Lattice QCD

Light Hadrons



Charmed Hadrons



N_f= 2+1, staggered sea-quarks, 16^3x48, 20^3x64, 28^3x96
a = 0.18, 0.12, 0.086 fm, L= 2.8, 2.4, 2.4 fm

MILC Coll., PoS (LAT2005) 203 [hep-lat/0510072]

Charmed hadron masses are also well reproduced in Lattice QCD

Lattice QCD Monte Carlo calculation

We can perform Lattice QCD Monte Carlo calculation of the Euclidean VEV of any operator *O* by taking its ensemble average as

$S_F(U)$:quark propagator

$$< 0 | O(A, q, \bar{q}) | 0 > = < O(U, S_F(U)) >$$

$$= \frac{\int DU \exp\{-S_{\mathrm{gauge}}(U)\} \det[S_F^{-1}(U)]\, O(U, S_F(U))}{\int DU \exp\{-S_{\mathrm{gauge}}(U)\} \det[S_F^{-1}(U)]}$$

$$= \lim_{a\to 0} \lim_{V\to\infty} \lim_{N\to\infty} \frac{\sum_{i=1}^{N} O(U_i, S_F(U_i))}{\sum_{i=1}^{N} 1}$$

:gauge ensemble average

Note here that the gauge ensemble generated by Monte Carlo method is the *important samples* of QCD gauge configuration. In fact, each obtained configuration represents *a huge number of ensembles*.

Two-point corelator in Euclidean QCD

We can extract ground-state and low-lying excited-state masses of hadrons with quantum number q from the large t behavor of Euclidean two-point correlator (Green's function) G_q of O_q.

$$G_q(t;\vec{p}) = < O_q(t;\vec{p}) | O_q(0;\vec{p}) >$$

$\vec{p}$:total 3-dim. momentum of hadron

$$= < O_q, \vec{p} | \exp(-Ht) | O_q, \vec{p} >$$

H:QCD Hamiltonian

$$= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} < 0 | O_q(t,\vec{x}) O_q(0,\vec{0}) | 0 >$$

$| O_q, \vec{p} >$:hadron state with total momentum ***p***

Taking ***p***=**0**, we have *zero-momentum-projected correlator*, which directly relates to the ground-state mass M_0 and excited-state masses M_n (n=1,2,..) as

$$G_q(t) \equiv G_q(t;\vec{p}=\vec{0}) = \sum_{\vec{x}} < 0 | O_q(t,\vec{x}) O_q(0,\vec{0}) | 0 > = \sum_{n=0}^{\infty} | c_n |^2 \exp(-M_n t)$$

Hadron mass calculation

The effective mass $M(t)$ is useful to extract hadron masses. In particular, the ground-state hadron mass M_0 can be obtained from the large t limit of $M(t)$.

$$G_q(t) = \sum_{\vec{x}} < 0 \,|\, O_q(t,\vec{x}) O_q(0,\vec{0}) \,|\, 0 > = \sum_{n=0}^{\infty} | c_n |^2 \exp(-M_n t)$$

$$M(t) \equiv \ln\{G_q(t) / G_q(t+1)\}$$

In large t limit, the effective mass $M(t)$ goes to the ground-state mass M_0 :

$$\lim_{t\to\infty} M(t) = M_0$$

Here, M_0 is the *lowest mass* of hadron with quantum number q.

In contrast to the lowest mass, it is *somewhat difficult* to extract the *excited-state masses* M_n (n=1,2,..).

Mass measurement of Excited-state hadrons

To extract the *excited-state masses* M_n (n=1,2,..), for example, we prepare many interpolating fields O^k (k=1,2,..), and calculate correlation matrix G^{ij}.

$$G^{ij}(t) = \sum_{\vec{x}} < 0 | O^i(t,\vec{x}) O^j(0,\vec{0}) | 0 >$$

$$= < O^i, \vec{p} = \vec{0} | e^{-Ht} | O^j, \vec{p} = \vec{0} > \qquad | O^k \rangle = \sum_{n=0}^{\infty} c_n^k | n \rangle$$

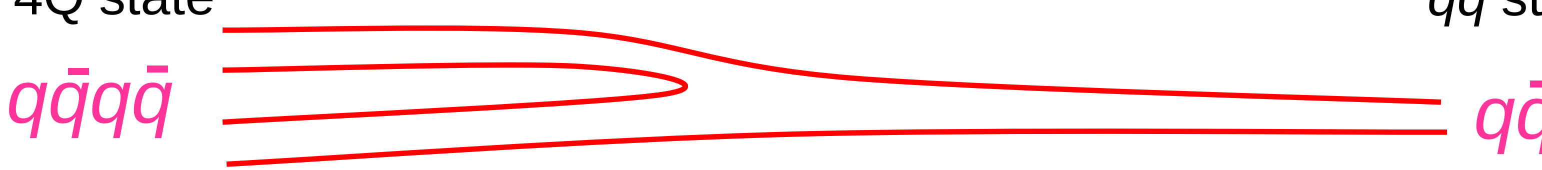
$$= \sum_{n=0}^{\infty} c_n^{i*} c_n^j \exp(-M_n t)$$

In principle, by diagonalize the correlation matrix G^{ij}, we can extract the *excited-state masses* M_n (n=1,2,..) from the diagonal elements, although this is rather tough work and not so easy to be performed with enough accuracy…

Difficulty of Multi-Quark calculations in Lattice QCD

Difficulty of Calculation on Multi-Quark States in Full QCD

In *Full Lattice QCD*, 4Q states with non-exotic quantum numbers automatically *mix with the $q\bar{q}$ component as a virtual intermediate state* due to the *quark-pair creation/annihilation*. Similarly, non-exotic 5Q states generally mix with 3Q states. *Then, it is almost impossible to single out "Multi-Quark component" without suffering from mixture of $q\bar{q}$ in full QCD.*



Full QCD

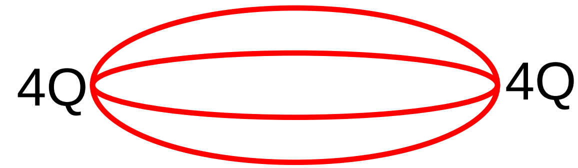
virtual intermediate state

⇒

$q\bar{q}$ state appears as virtual state in full QCD

Possible Calculation of 4Q or 5Q state in Quenched QCD

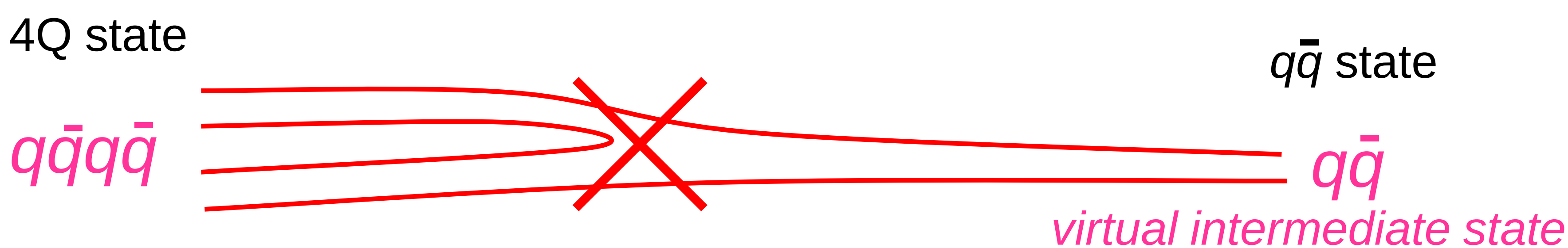
Note that, in *Quenched Lattice QCD*, the *4Q state component* can be investigated *without* suffering from the *mixture of $q\bar{q}$ component*, because there is *no* dynamical quark-pair annihilation and creation in quenched QCD.



Similarly, *5Q state component* can be investigated *without 3Q mixture* in quenched QCD.

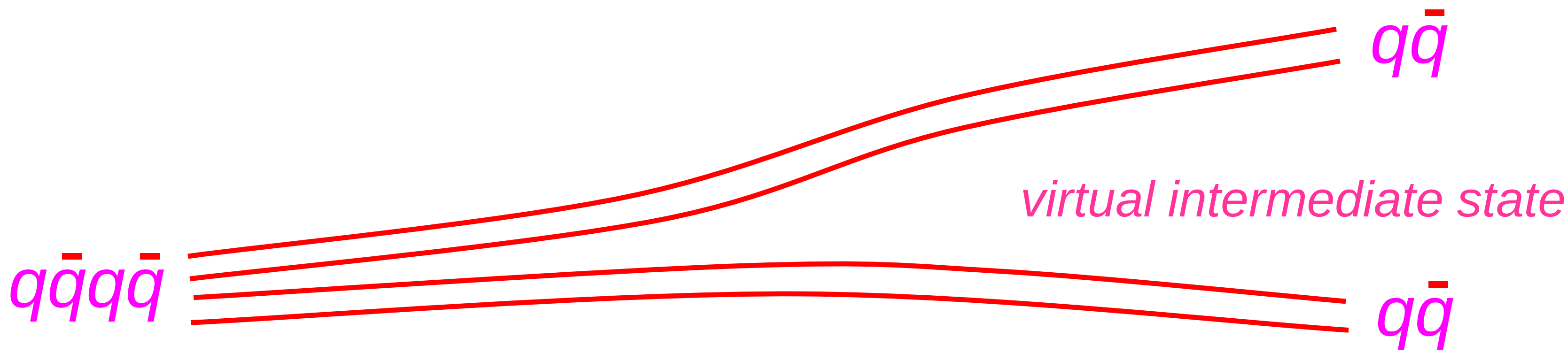
~ a sort of “merit” of quenched approximation
for the study of multi-quark hadrons

In fact, each *Multi-Quark component* can be investigated individually in *quenched QCD*.



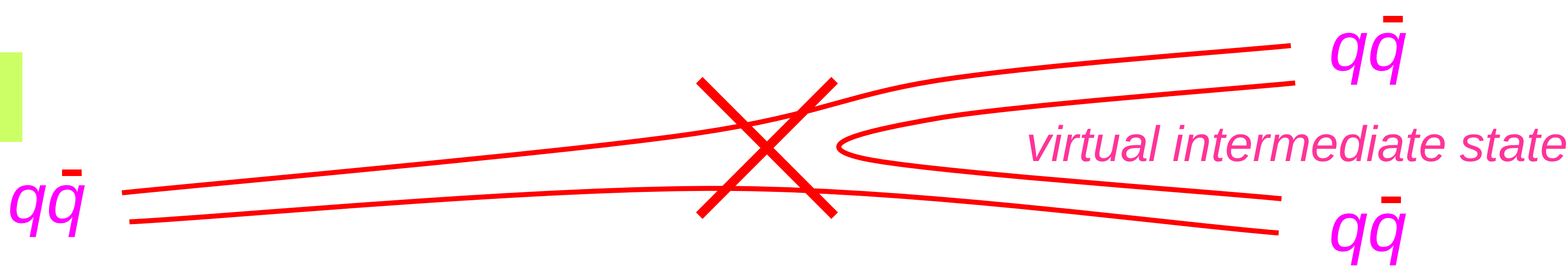
Quenched QCD forbids quark-pair annihilation and creation

Nevertheless, there remains a problem on the “possible decay” of Multi-Quark Hadrons even in quenched QCD



In fact, Tetra- and Penta-quarks can *decay into two hadrons*, and therefore multi-quark hadrons are unstable even in quenched QCD.

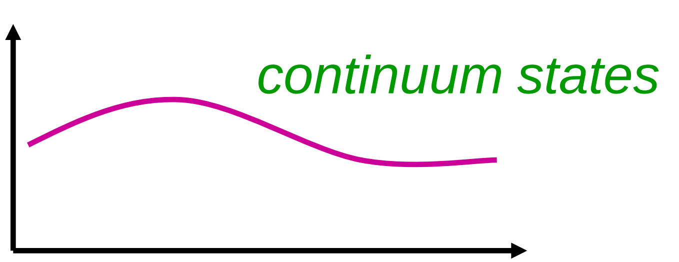
cf.

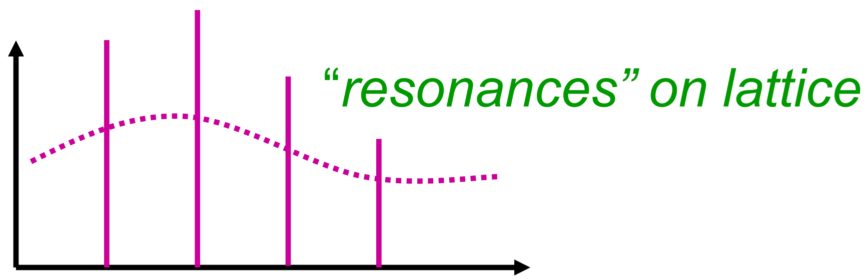


In contrast, ordinary hadrons cannot decay into two hadrons in quenched QCD, because such a decay accompany q-$\bar{q}$ pair creation. In fact, ordinary hadrons are stable in quenched QCD.

Difficulty of Lattice QCD Analysis for Multi-Quark Hadrons

- In general, it is rather *difficult* to perform the *Lattice QCD Analysis* for *multi-quark hadrons*.
- This is basically because the *multi-quark hadrons appear as excited states and can be mixed with two hadron states*.
- In fact, it is rather *difficult to distinguish* the *multi-quark resonances* from *two hadron scattering states* in lattice QCD.
- Moreover, on a *finite-volume lattice*, the *two hadron scattering states* appear as *"resonance-like states"* instead of *continuum states* due to the *momentum discretization*. This makes difficult to distinguish the multi-quark hadrons from the scattering states.





The experimental status of pentaquark baryon Θ^+(1530) is not so clear, since Prof. Nakano's recent new analysis indicates its existence again.

The present status of pentaquark is still **controversial** also in lattice QCD!

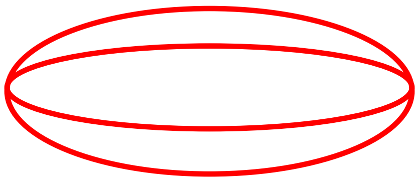
1. F.Csikor, Z.Fodor et al. JHEP 0311 (03) 070 → JHEP (05)
   negative-parity 1/2 pentaquark near KN threshold → **No pentaquarks**
2. S.Sasaki PRL93 (04), **negative-parity 1/2 pentaquark** near KN threshold
3. N.Mathur et al. (Kentucky group) Lattice 04, PRD70(04)
   No spin-1/2 pentaquark near KN threshold in both parity channels
4. N. Ishii, H.S. et al. Lattice 04, PRD71(05), PRD72(05),
   No spin 1/2, 3/2 light pentaquark resonance below 1.75GeV in both parity
5. T.T.Takahashi et al.(YITP group), PRD71(05),
   negative-parity spin-1/2 pentaquark about 1.8GeV
6. B.G.Lasscock et al. (Adelaide group), PRD72 (05), PRD(06)
   No spin-1/2 pentaquark, **spin-3/2 pentaquark** about 1GeV??
7. T.-W. Chiu et al., PRD72 (05) **positive-parity spin-1/2** pentaquark

Lattice QCD Studies for Light Tetra-Quarks

Tetraquark simulations of light scalars

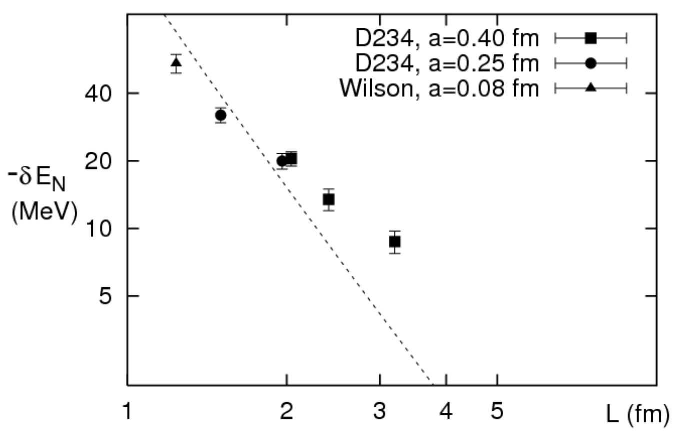
Alford and Jaffe (2000)

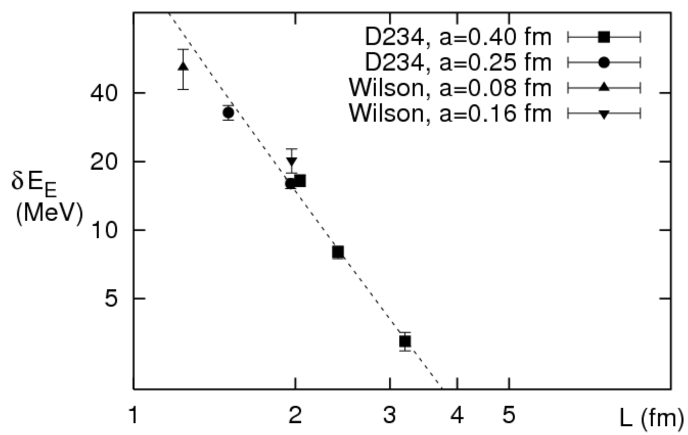
- **π** -**π** type interpolating field for σ: $(\bar{q}\gamma_5 q)(\bar{q}\gamma_5 q)$
- relatively heavy quark
- quenched, analysis of connected diagram
- check on volume dependence



They compare lattice data with the relation for scattering at finite volume (Luscher 1986)

$$\delta E_\alpha = E_\alpha - 2m_P = \frac{T_\alpha}{L^3}\left(1 + 2.8373\,\frac{m_P T_\alpha}{4\pi L} + 6.3752\left(\frac{m_P T_\alpha}{4\pi L}\right)^2 + \cdots\right),$$





- non-exotic channel, I=0 of $SU(2)_{flavor}$
- interpreted as repulsiveπ -π scattering
  + possible tetraquark resonance

- exotic channel, I=2 of $SU(2)_{flavor}$
- interpreted as repulsiveπ -π scatt.

Possible indication of a tetraquark resonance ??

Tetraquark simulations of light scalars

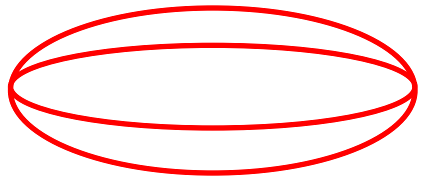
H.S, K.Tsumura, N.Ishii, F.Okiharu Prog.Thor.Phys.Suppl.(2006)

-non **π** -**π** type (diquark-type) interpolating field:

$$O_{4Q} \equiv \epsilon_{abc}\,\epsilon_{ade}\,(d_b^T\,C\,\gamma_5\,u_c)\,(\bar{u}_d\,\gamma_5\,C\,\bar{d}_e^T),$$

-relatively heavy quark (m_s~$2m_s$, i.e., $m_\pi \geqq$ 650MeV)

-quenched, analysis of 4Q connected diagrams



-*O*(*a*)-improved action (clover fermion)

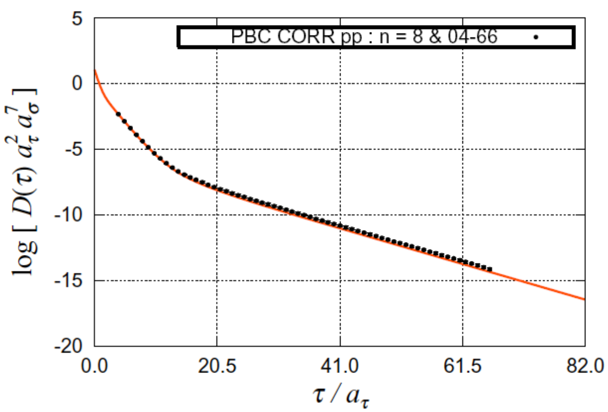
-large statistics (about 2000 gauge configurations)

-Dirichlet boundary condition on quark field in temporal direction
to avoid the contamination of backward propagations.

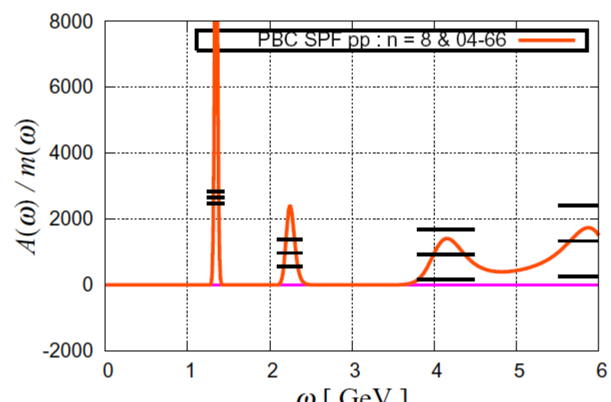
-**anisotropic** lattice (**high-resolution** in temporal direction)

-check on **boundary-condition dependence** (PBC vs **Hybrid BC**)

-**MEM analysis** to obtain the **spectral function A(ω) from correlator D(τ)**



⇒



O(a)-improved Anisotropic Lattice QCD

Gauge part $$S_G = \frac{\beta}{N_c}\frac{1}{\gamma_G}\sum_{x,i<j\leq 3}\mathrm{ReTr}\{1-P_{ij}(x)\} + \frac{\beta}{N_c}\gamma_G\sum_{x,i\leq 3}\mathrm{ReTr}\{1-P_{i4}(x)\}$$

Quark part : clover fermion

~ O(a)-improved i.e. discretized error is reduced

$$S_F \equiv \sum_{x,y}\bar{\psi}(x)K(x,y)\psi(y),$$

(a: lattice spacing)

$$\begin{aligned} K(x,y) \equiv\ & \delta_{x,y} - \kappa_t\left\{(1-\gamma_4)U_4(x)\delta_{x+\hat{4},y} + (1+\gamma_4)U_4^\dagger(x-\hat{4})\delta_{x-\hat{4},y}\right\} \\ & -\kappa_s\sum_i\left\{(r-\gamma_i)U_i(x)\delta_{x+\hat{i},y} + (r+\gamma_i)U_i^\dagger(x-\hat{i})\delta_{x-\hat{i},y}\right\} \\ & -\kappa_s c_E\sum_i \sigma_{i4}F_{i4}\delta_{x,y} - r\kappa_s c_B\sum_{i<j}\sigma_{ij}F_{ij}\delta_{x,y}, \end{aligned}$$

β=5.75 on $12^3 \times 96$ with renormalized anisotropy $a_s/a_t=4$

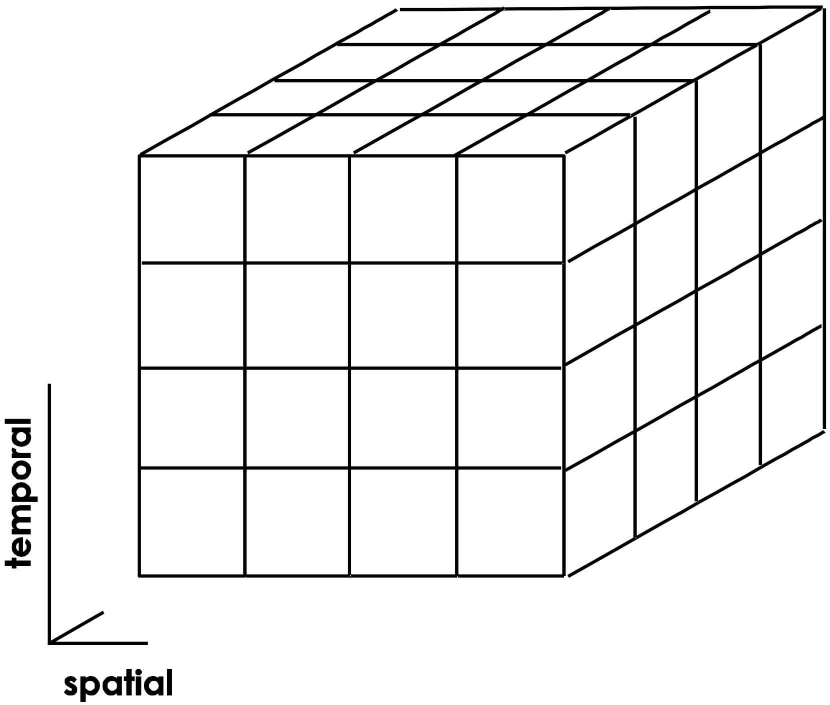
This leads to the spatial/temporal lattice spacing as
a_s = 0.18fm (a_s^{-1} = 1.10GeV), a_t = 0.045fm (a_t^{-1}= 4.40GeV).
The spatial lattice size is L =12 a_s = 2.15 fm. (Sommer scale: r_0^{-1}=395MeV)

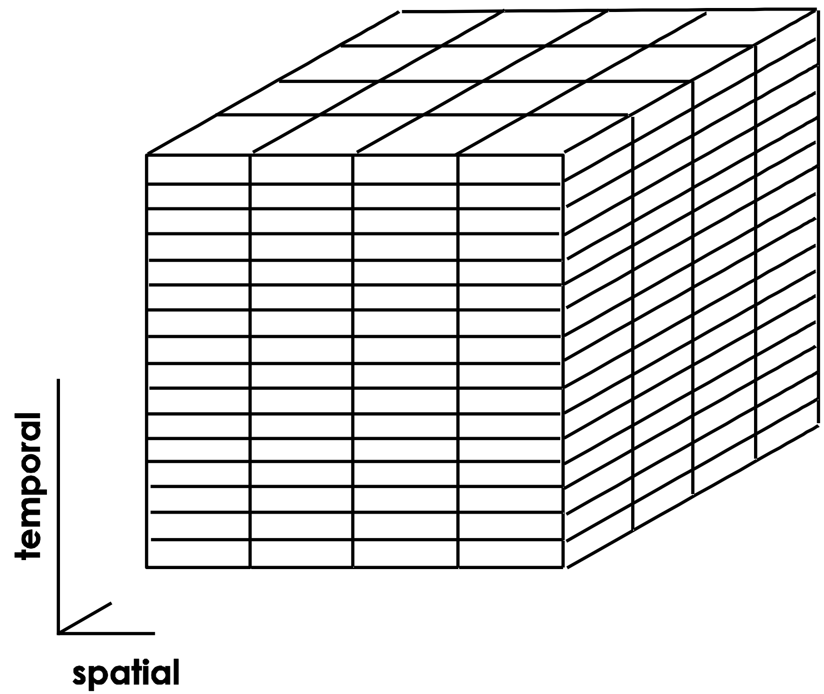
happing parameter: **κ** =0.1210 ~ 0.1240 (m_s~2m_s,) → $m_\pi \geqq$ 650MeV.

1,827 gauge configurations, picked up every 500 sweeps after thermalization of 10,000 sweeps.

Anisotropic Lattice QCD

To get detailed information on the temporal behavior of the 4Q correlator $D_{4Q}(\tau)$, we adopt anisotropic lattice QCD with high-resolution in the temporal direction.





4Q correlator and effective mass analysis

From the temporal 4Q correlator D_{4Q} (t), the low-lying mass of the 4Q state is estimated by the "effective mass"

$$m_{\mathrm{eff}}(t) = \ln \{D_{4Q}(t)/D_{4Q}(t+1)\}$$

in large *t* region.

- Zero-momentum projection is done to remove the total kinetic energy.
- We use the same quark mass for u and d.
- We use spatially-extended operators to enhance the low-lying spectra.
- For temporal direction, we impose Dirichlet boundary condition on the quark fields to avoid the contamination of backward propagations.
- For spatial directions, we use not only standard periodic boundary condition (PBC) but also the hybrid boundary condition (HBC), where $q\bar{q}$ meson state inevitably has a finite momentum while 4Q state can have zero momentum.

Hybrid Boundary Condition (HBC)

~New method to distinguish resonance and scattering state

Table I. Hybrid Boundary Condition (HBC) to raise the threshold of two-meson scattering states.

	u, d	$\bar{u}$, $\bar{d}$	$q\bar{q}$-meson	two-meson threshold	tetra-quark ($qq\bar{q}\bar{q}$)
PBC	periodic	periodic	periodic	$m_1 + m_2$	periodic
HBC	anti-periodic	periodic	anti-periodic	$\sum_{k=1,2} \sqrt{m_k^2 + \vec{p}_{\min}^2}$	periodic

In the Hybrid Boundary Condition (HBC) method,
anti-periodic and periodic boundary conditions are imposed on quarks and anti-quarks, respectively.
By applying the HBC on a finite-volume lattice,
the mass of $q\bar{q}$ meson state is raised up, while the mass of spatially compact 4Q resonance is almost unchanged.

cf. Through the HBC test, low-lying 5Q is clarified to be a NK scattering state.

No low-lying Penta-Quark resonances ($1/2^+$, $1/2^-$, $3/2^+$, $3/2^-$)

N.Ishii et al. Proc. of Lattice 2004 before null-result experiments reported

Physical Review D71 (2005) 034001,Physical Review D71 (2005) 074503

Hybrid Boundary Condition (HBC)

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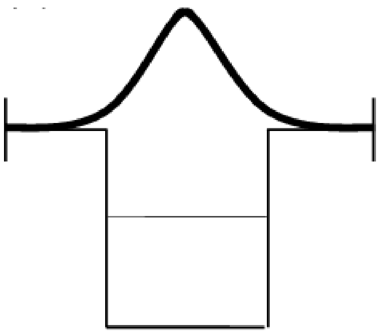
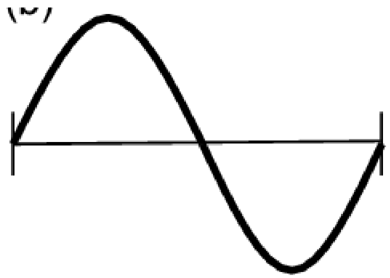
HBC	anti-periodic	periodic	anti-periodic	$\sum_{k=1,2} \sqrt{m_k^2 + \vec{p}_{\min}^2}$	periodic

- In fact, ***spatially compact resonance*** is ***not*** so *sensitive to the spatial boundary condition*.



spatially compact resonance

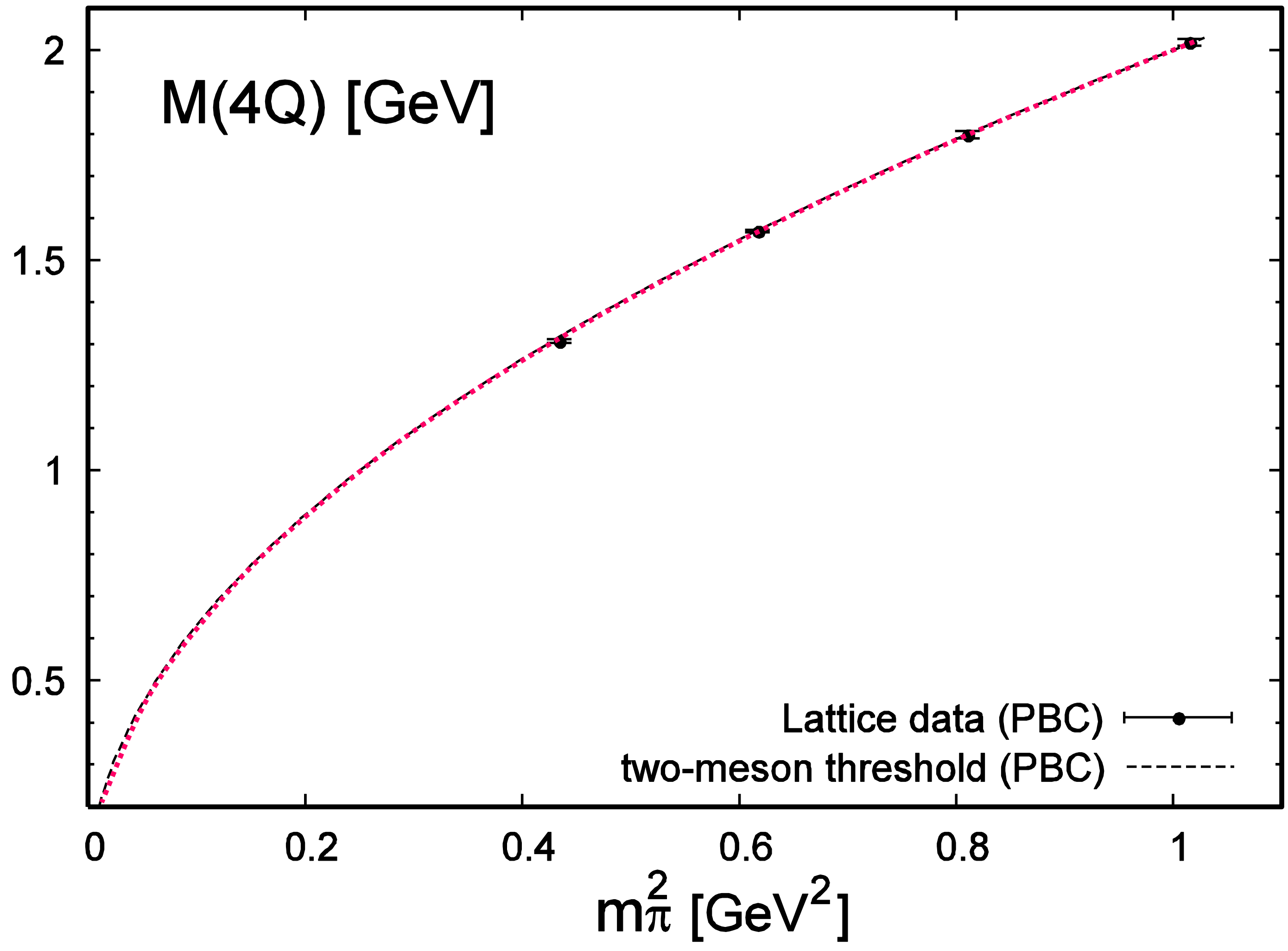
- In contrast, ***scattering states***, which are spatially extended, are rather *sensitive to the spatial boundary condition*.



scattering state

We can distinguish resonance and scattering state scattering states by examining the sensitivity on the spatial boundary condition.

The mass of lowest 4Q state from effective-mass analysis in lattice QCD



The lowest 4Q state almost coincides with the **π π** threshold.
The lowest 4Q is a **π π** scattering state. This is natural but trivial.

MEM analysis to obtain the spectral function from the temporal correlator

To clarify whether the resonance state exits or not in the low-lyong 4Q spectrum, we perform ***MEM analysis*** to *obtain the **spectral function** A(ω) from the **temporal correlator** D(τ)*, and investigate its peak structure and boundary-condition dependence.

MEM analysis also reveals the possible surviving of J/ψ even above the critical temperature T_c in lattice QCD.

[Asakawa-Hatsuda, Datta et al., Umeda et al.,
H.Iida, T.Doi, N.Ishii, H.S., K.Tsumura, PRD74 (2006) 074502.]

Maximum Entropy Method (MEM)

MEM is a useful method to obtain the spectral function A(ω) in a model-independent way from the correlator $D(\tau)$ by solving the inverse problem,

$$D(\tau) = \int_0^\infty \mathrm{d}\omega\, K(\tau\,,\,\omega)\, A(\omega),$$

where the kernel at zero temperature is given by $K(\tau,\omega) \equiv e^{-\tau\omega}$.

Generally, the inverse problem is mathematically difficult, and it is actually difficult to solve the linear simultaneous equation directly with respect to A(ω) both analytically and numerically.

In the MEM framework, for the given lattice data of the correlator $D(\tau)$, the most probable spectral function A(ω) is extracted, by maximizing the conditional probability $P[A|D]$ of A(ω) for given $D(\tau)$, instead of solving the linear simultaneous equation directly with respect to A(ω).

Using the *Bayes theorem* in probability theory,

$$P[A|D] = P[D|A]\, P[A] \,/\, P[D],$$

the conditional probability $P[A|D]$ is obtained as a functional with respect to A(ω),

$$P[A|D] \propto e^{\alpha S[A] - L[A]}$$

- Here, L[A] is the *likelihood function*,

$$L[A] = \frac{1}{2} \sum_{i,j} \Big(D(\tau_i) - D[A](\tau_i)\Big)\, C_{ij}^{-1} \Big(D(\tau_j) - D[A](\tau_j)\Big),$$

whose minimization gives the ordinary χ^2 fit.

($D[A](\tau) \equiv \int d\omega K(\tau,\omega)\, A(\omega)$, C^{-1}: inverse covariant matrix)

- S[A] is the *Shannon-Jaynes entropy* defined as

$$S[A] = \int d\omega \left[A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right],$$

where $m(\omega)$ is the *default function* to give a plausible form of A[ω], e.g., P-QCD form in the high-energy region.

In MEM, L[A] *tends to be minimized and S*[A] *to be maximized.*

Default Function of 4Q state

Default function $m(\omega)$ appearing in the Shannon-Jaynes entropy S[A] gives a plausible form of the spectral function A[ω],
e.g., P-QCD form in the high-energy region,
which plays a role of a suitable "boundary condition" of A[ω] at large ω.

For the local 4Q operator, $O_{4\mathrm{Q}} \equiv \epsilon_{abc}\,\epsilon_{ade}\,(d_b^T\,C\,\gamma_5\,u_c)\,(\bar{u}_d\,\gamma_5\,C\,\bar{d}_e^T),$
we calculate the default function $m(\omega)$ of the 4Q state at the lowest perturbation of QCD as

$$m(\omega) = 4\,N_C \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}$$
$$\delta(\omega - E(\boldsymbol{k}) - E(\boldsymbol{p}) - E(\boldsymbol{q}) - E(\boldsymbol{k}+\boldsymbol{p}-\boldsymbol{q}))\left[1 - \frac{\boldsymbol{k}\cdot\boldsymbol{p} - m^2}{E(\boldsymbol{k})E(\boldsymbol{p})}\right]\left[1 - \frac{\boldsymbol{q}\cdot(\boldsymbol{k}+\boldsymbol{p}-\boldsymbol{q}) - m^2}{E(\boldsymbol{q})E(\boldsymbol{k}+\boldsymbol{p}-\boldsymbol{q})}\right]$$

$$E(\boldsymbol{p}) \equiv \sqrt{|\boldsymbol{p}|^2 + m^2}.$$

and we get the final form as

$$\boxed{m(\omega) = m_0\,\omega^8 \equiv \frac{4\,N_C}{2^8\,\Gamma(5)\,\Gamma(6)\,\pi^6}\,\omega^8}$$

MEM analysis for the 4Q state

From the lattice QCD data for the 4Q correlator $D_{4Q}(x) = \langle\, O_{4Q}(x)\, O_{4Q}(0)\,\rangle$

with the *local 4Q operator* $O_{4Q} \equiv \epsilon_{abc}\,\epsilon_{ade}\,(d_b^T\, C\,\gamma_5\, u_c)\,(\bar{u}_d\,\gamma_5\, C\,\bar{d}_e^T),$

we get the *spectral function A(ω) for 4Q state* through the *MEM analysis*.

Note that the *appearance of the peak structure* in the spectral function A(ω) in the MEM analysis is *highly nontrivial*,

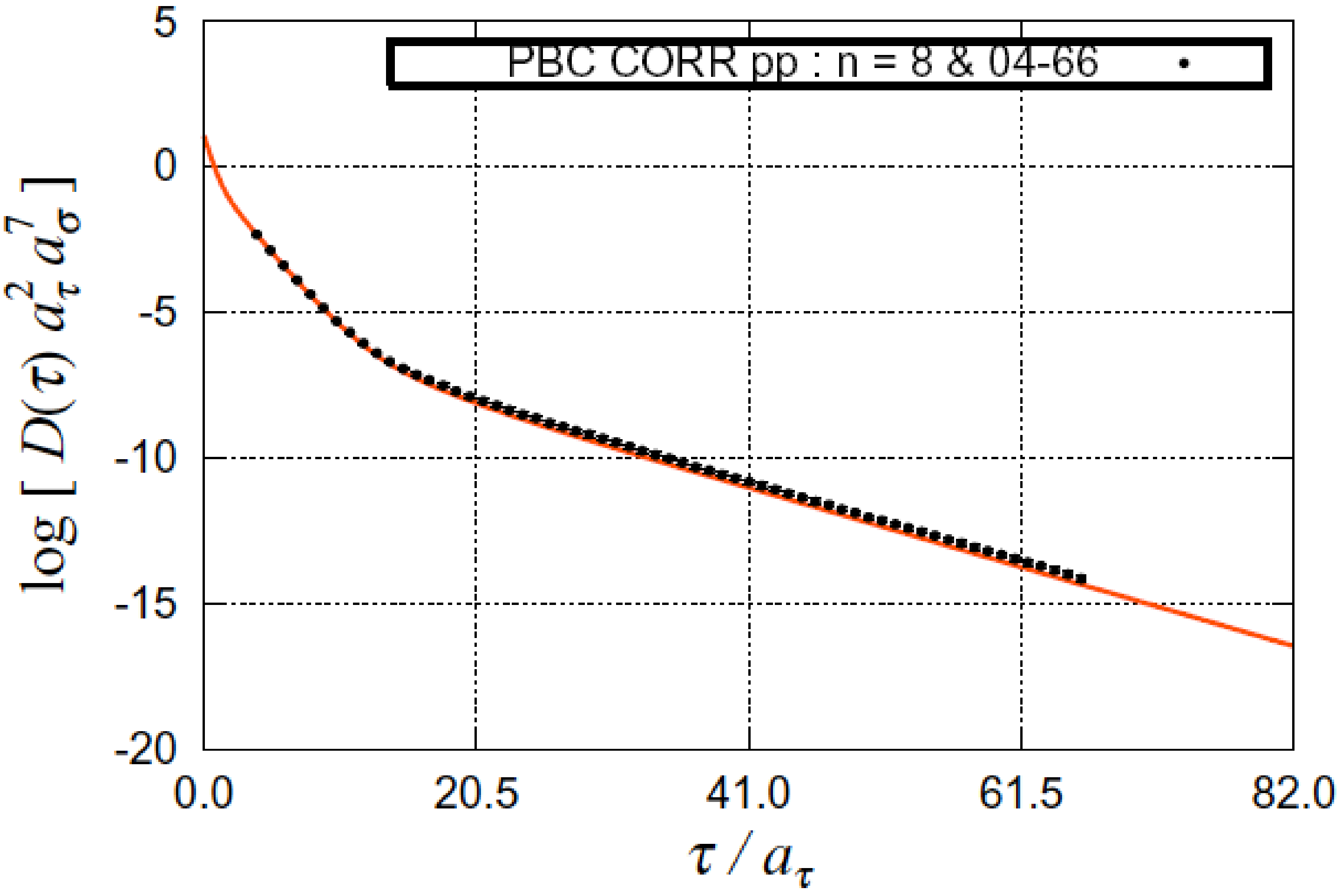
because the *default function* *m*(ω) obtained with P-QCD
*does **not** have any peak structure*.

$$m(\omega) = m_0\,\omega^8 \equiv \frac{4\,N_C}{2^8\,\Gamma(5)\,\Gamma(6)\,\pi^6}\,\omega^8$$

~ *simple monotonic function*

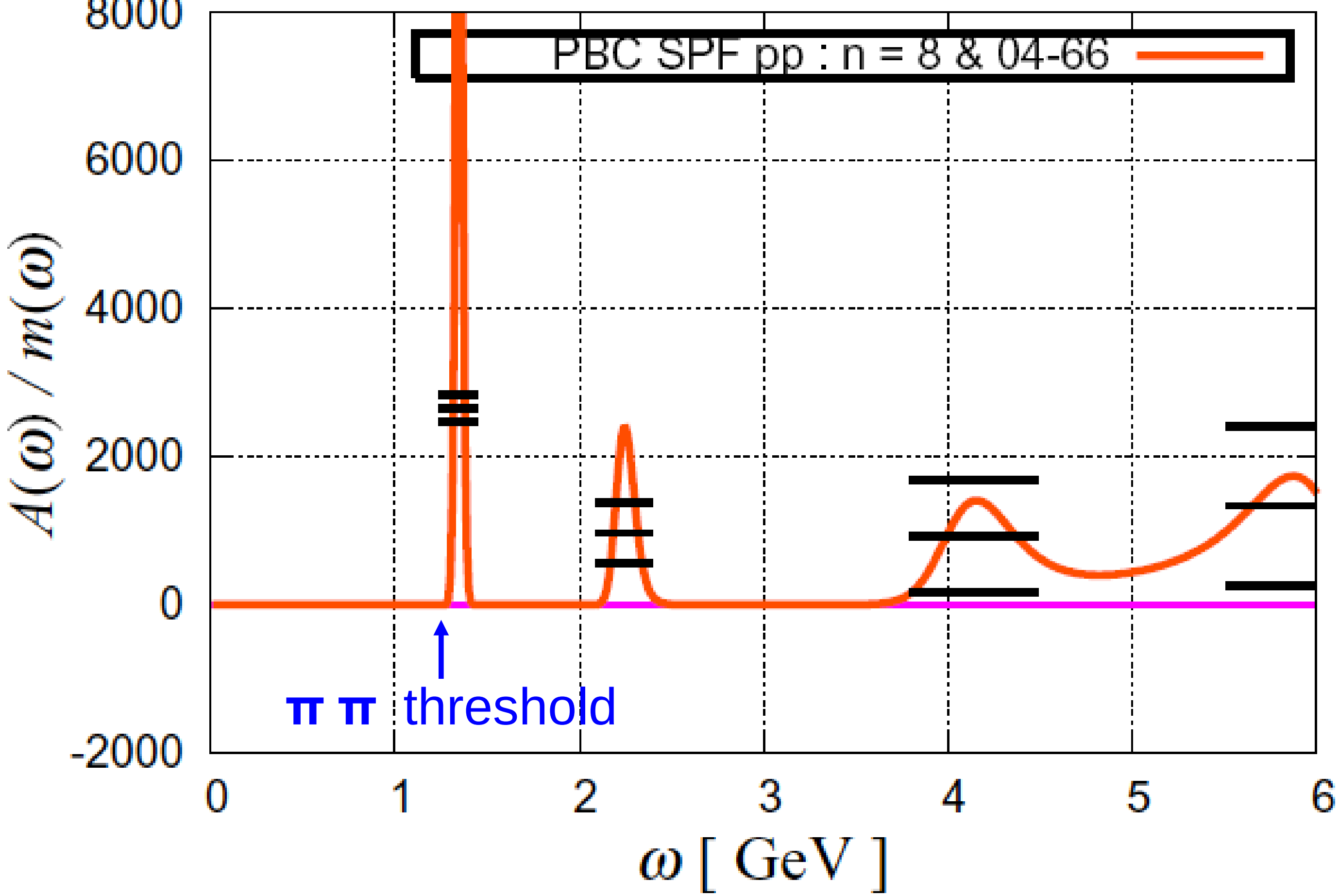
With this default function, the peak structure of the spectral function A(ω)
is biased to be disappeared in the MEM analysis.
→ The peak structure surviving in the MEM analysis is mathematically reliable.

Lattice data for 4Q correlator (PBC) and MEM result



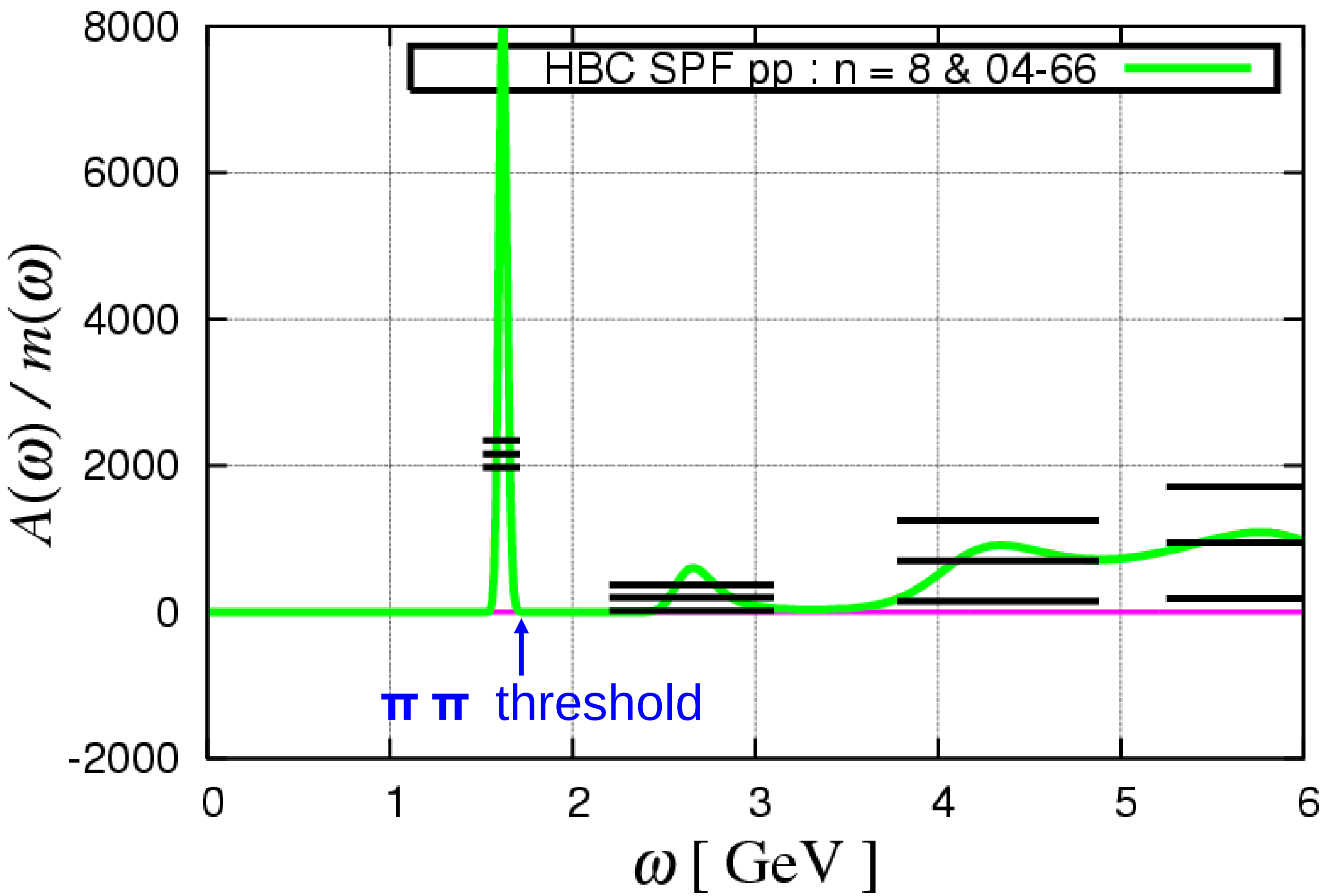
The lattice data for 4Q correlator *D*(τ) is well reproduced with the spectral function A(ω) obtained by MEM

Spectral function for 4Q state (PBC) obtained with MEM



Sharp peak appears just above **π π** threshold (~1.3GeV)
→ resonance state ? But, in finite-volume lattice,
the momentum of the pion is discretized, and
all the scattering states are observed as resonance-like states.

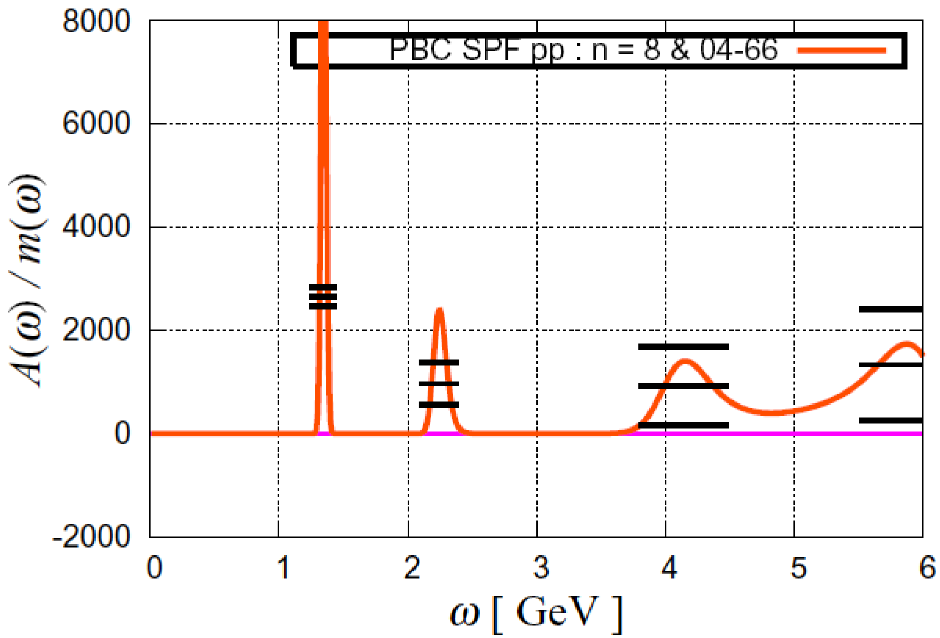
Spectral function for 4Q state (HBC) obtained with MEM



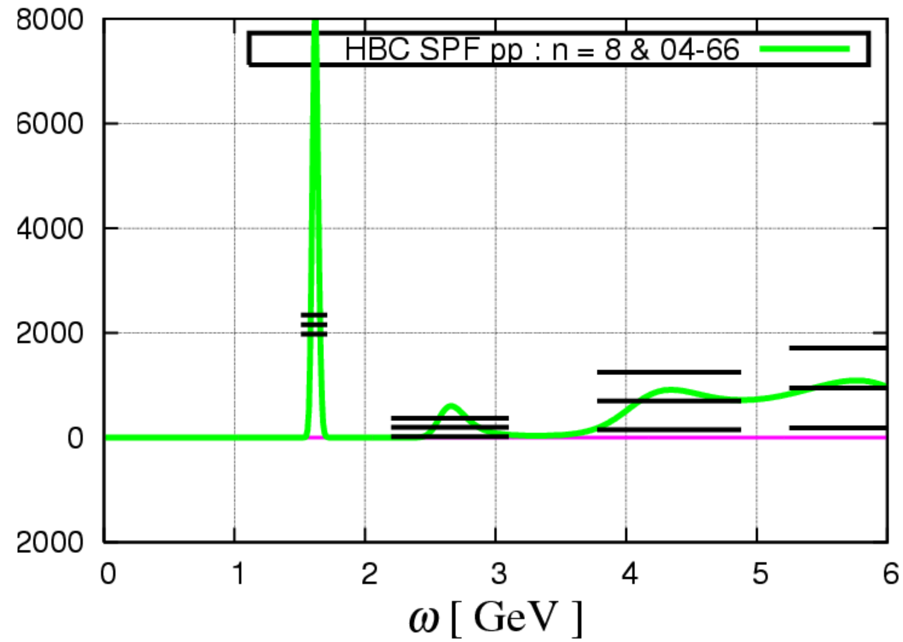
Sharp peak appears just below HBCπ π threshold (~1.7GeV)
The peak position is shifted → resonance state sensitive to BC
~ spatially-extended two-pion scattering state

Spectral function for 4Q state obtained with MEM

Ordinary Periodic BC



Hybrid BC



The peak position is shifted → resonance state sensitive to BC

~ spatially-extended two-pion scattering state

There is no extra low-lying peak in PBC

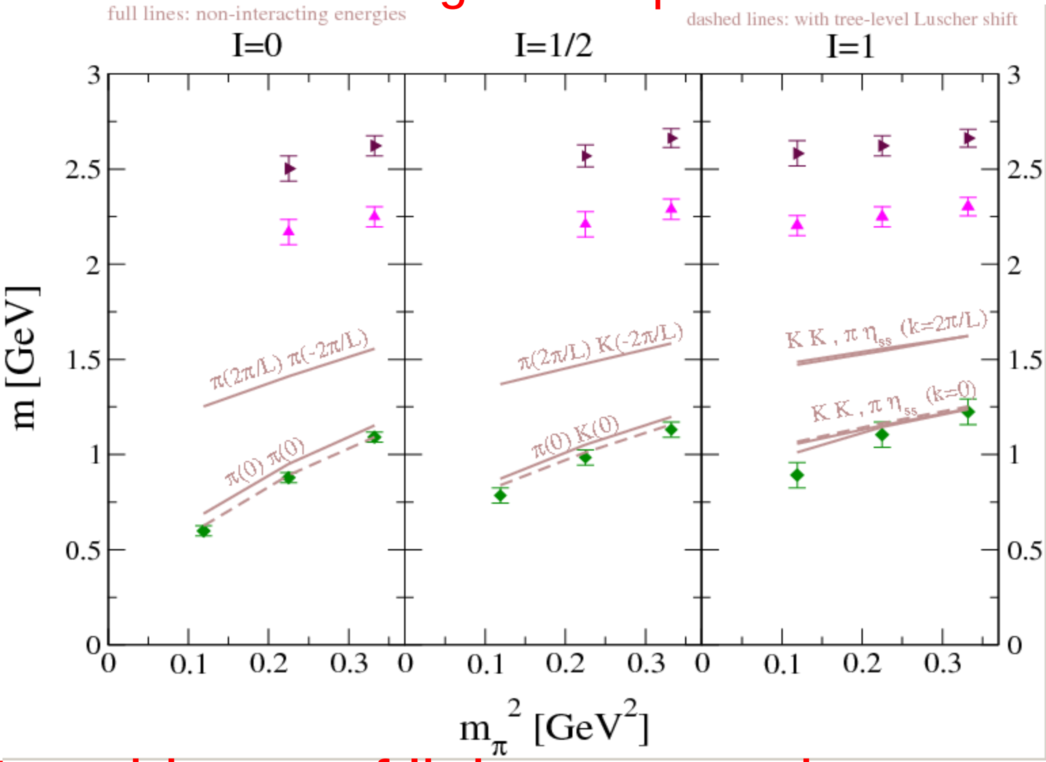
→ No low-lying four-quark resonance state

Tetraquark simulations of light scalars with I=0,1/2,1

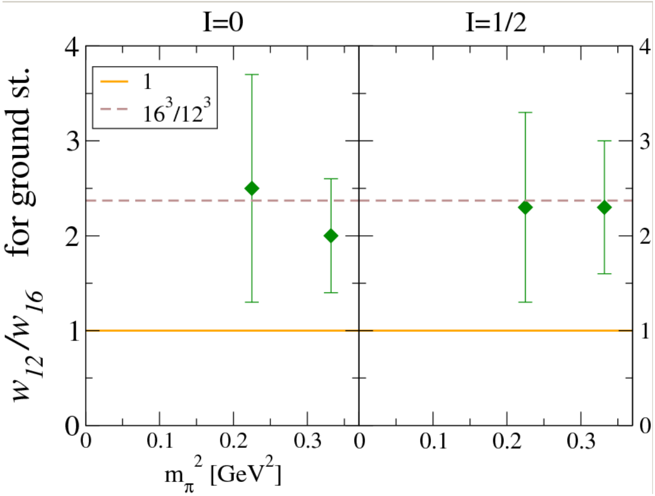
Sasa Prelovsek, Lattice 2008

- ground state: effective mass and volume dependence of spectral weights roughly consistent with tower of scattering states
- excited states: too heavy to correspond to light tetraquark candidates ~they may be also scattering states with finite momentum

→ No evidence of light tetraquark resonances for m_π = 340-570MeV



check on spectral weight



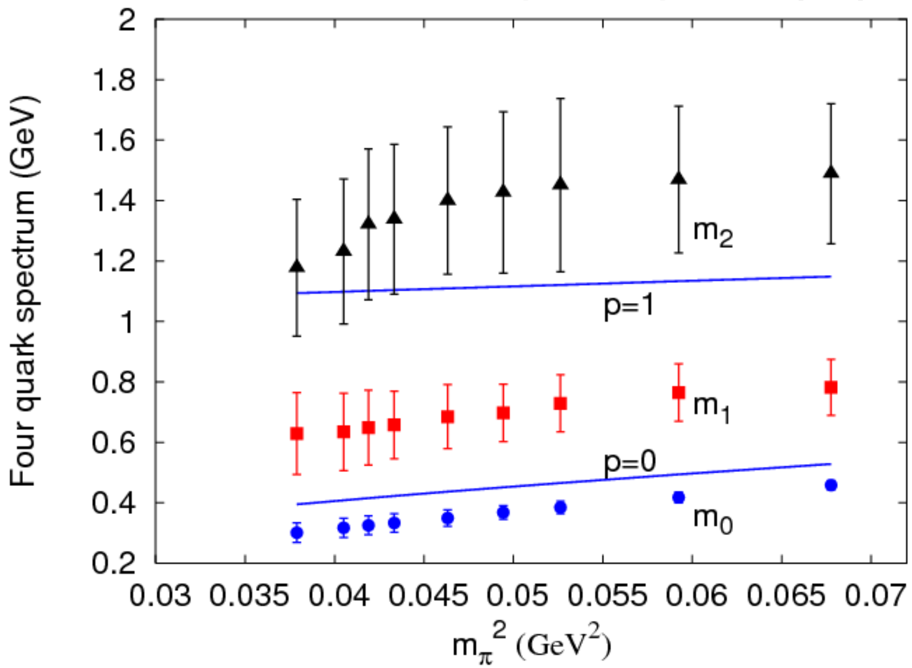
No evidence of light tetraquark resonances for m_π = 340-570MeV.

Tetraquark simulations of light scalars

N.Mathur, K.-F.Liu et al. (Kentucky Group), hep-ph/0607110, PRD(2006)

-π -π type interpolating field for σ: $(\bar{q}\gamma_5 q)(\bar{q}\gamma_5 q)$

- small u,d quark masses with overlap-fermion
- quenched
- volume dependence of spectral weight to distinguish between resonance (W~L^0) and pi-pi scattering (W~1/L^3):

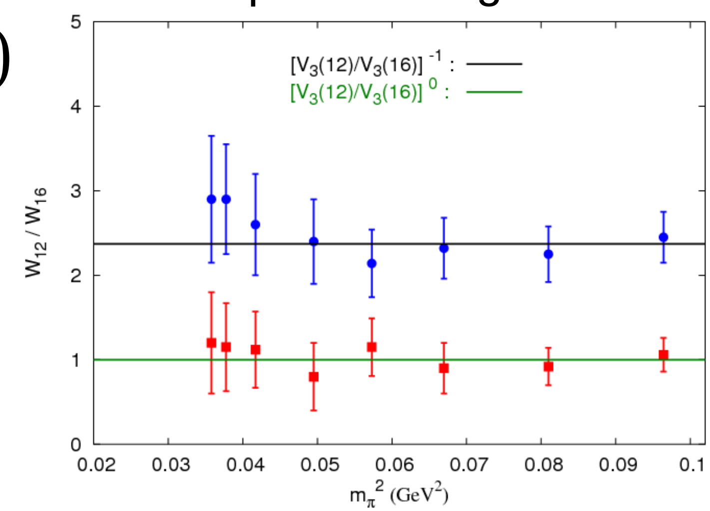


$\pi(1)\pi(-1)$

σ(600)?

$\pi(0)\pi(0)$

volume dependence check of spectral weight



They find σ(600)? as a tetraquark resonance near chiral limit, m_π <300MeV → but need to be checked

Multi-Quark Potential in Lattice QCD

A solid new lattice result
motivated by multi-quark

~other possible approach combined with quark model calculation

Strategy to understand hadron properties from QCD

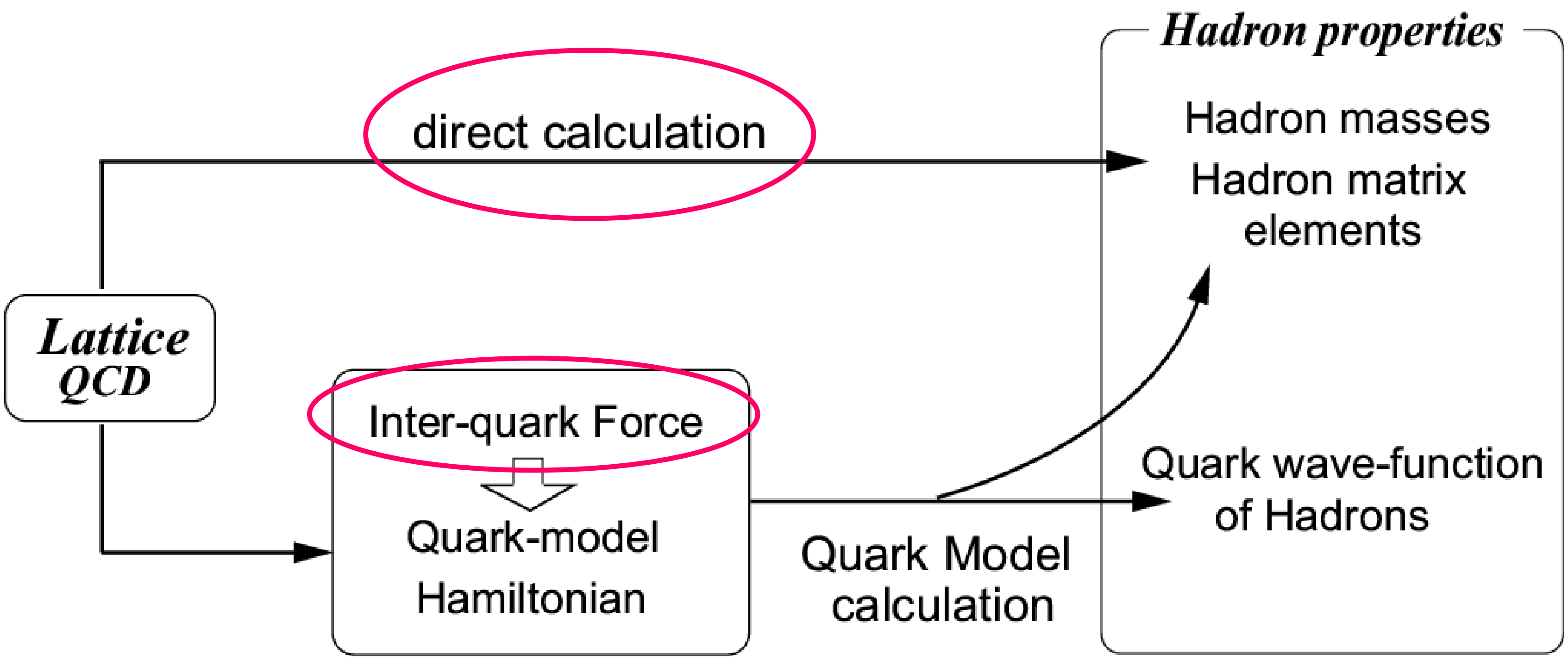


Figure 1: Our global strategy to understand the hadron properties from QCD. One way is the direct lattice QCD calculations for the low-lying hadron masses and simple hadron matrix elements, although the wave function is unknown and the practically calculable quantities are severely limited. The other way is to construct the quark model from QCD. From the analysis of the inter-quark forces in lattice QCD, we extract the quark-model Hamiltonian. Through the quark model calculation, one can obtain the quark wave-function of hadrons and more complicated properties of hadrons including properties of excited hadrons.

Multi-Quark Hadrons and Multi-Quark Potentials

Obviously, the *quark-model calculation* is one of the basic and standard theoretical methods to investigate multi-quark systems.

Note here that the quark-model calculation gives the *quark wave-function* as an important state information, while lattice QCD based on the path-integral formalism cannot gives the wave-function in a simple way.

In fact, the quark model calculation gives an outline of the *properties* of multi-quark hadrons and may *predict new-type exotic hadrons* theoretically.

But, for the quark-model calculation of multi-quarks, we have to give the *quark-model Hamiltonian* for multi-quark system, namely, the *multi-quark potential*, since the kinetic part is trivial.

We perform first study of multi-quark potential in lattice QCD.

Inter-quark potential in QCD

In 1979, M.Creutz performed the first application of lattice QCD simulation for the quark-antiquark potential using the Wilson loop.

Since then, the study of the inter-quark force has been one of the central issues in lattice QCD.

Actually, in hadron physics, the inter-quark force can be regarded as an elementary quantity to connect the “quark world” to the “hadron world”, and plays an important role to hadron properties.

In 1999, in addition to the quark-antiquark potential, we performed the first accurate reliable lattice QCD study for the three-quark (3Q) potential, which is responsible to the baryon structure at the quark-gluon level.

Furthermore, in 2005, we performed the first lattice QCD study for the multi-quark potentials, i.e., 4Q and 5Q potentials, which give essential information for the multi-quark hadron physics.

Note also that the study of 3Q and multi-quark potentials is directly related to the quark confinement properties in baryons and multi-quark hadrons.

Systematical Studies for Multi-Quark Potential in Lattice QCD

"Detailed Analysis of Tetraquark Potential and Flip Flop in SU(3) Lattice QCD"
F. Okiharu, H. Suganuma and T.T. Takahashi
Physical Review D72 (2005) 014505 (17 pages).

"First Study for the Pentaquark Potential in SU(3) Lattice QCD"
F. Okiharu, H. Suganuma and T.T. Takahashi
Physical Review Letters 94 (2005) 192001 (4 pages).

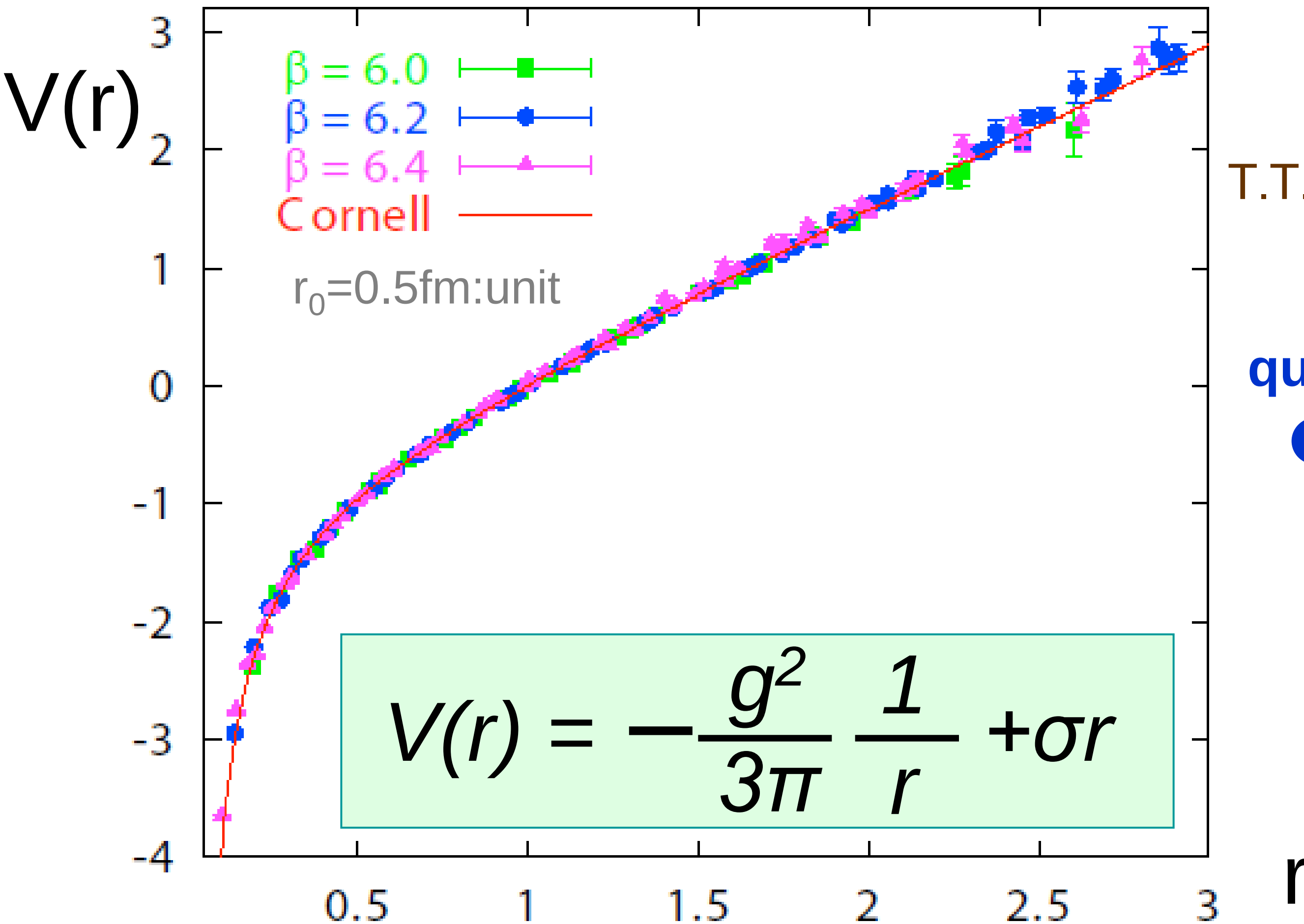
"Detailed Analysis of the Gluonic Excitation in the 3Q System in Lattice QCD"
T.T. Takahashi and H. Suganuma
Physical Review D70 (2004) 074506 (13 pages).

"Gluonic Excitation of the Three-Quark System in SU(3) Lattice QCD"
T.T. Takahashi and H. Suganuma
Physical Review Letters 90 (2003) 182001 (4 pages).

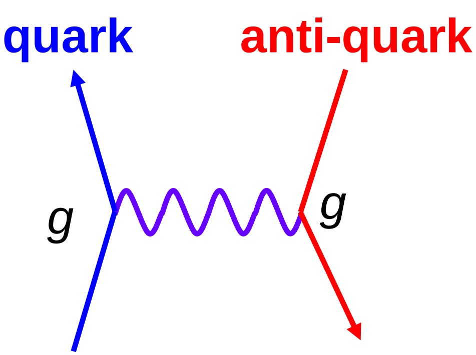
"Detailed Analysis of the Three Quark Potential in SU(3) Lattice QCD"
T.T. Takahashi, H. Suganuma et al.
Physical Review D65 (2002) 114509 (19 pages).

"Three-Quark Potential in SU(3) Lattice QCD"
T.T. Takahashi, H. Suganuma et al.
Physical Review Letters 86 (2001) 18-21.

Quark-antiquark static potential in Lattice QCD



M.Creutz (1979,80)
G.S.Bali (2001)
T.T.Takahashi et al. (2002)
JLQCD (2003)

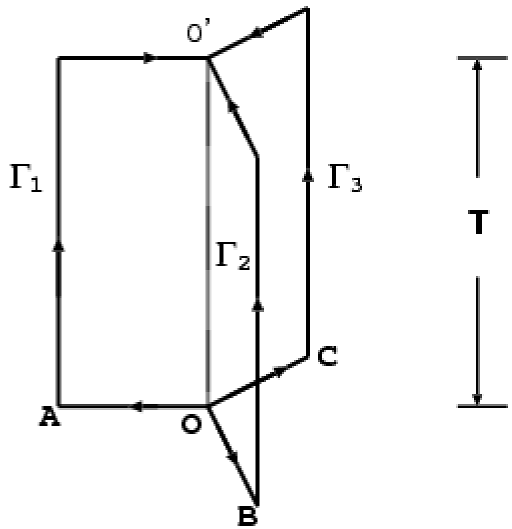


One-dimensional squeezing of color flux between q and $\overline{q}$

Similar to the Q-$\bar{\text{Q}}$ potential calculated with the Wilson loop, the 3Q potential can be calculated with the 3Q Wilson loop defined on the contour of three large staples as

$$W_{3\mathrm{Q}} \equiv \frac{1}{3!}\epsilon_{abc}\epsilon_{a'b'c'}U_1^{aa'}U_2^{bb'}U_3^{cc'}$$

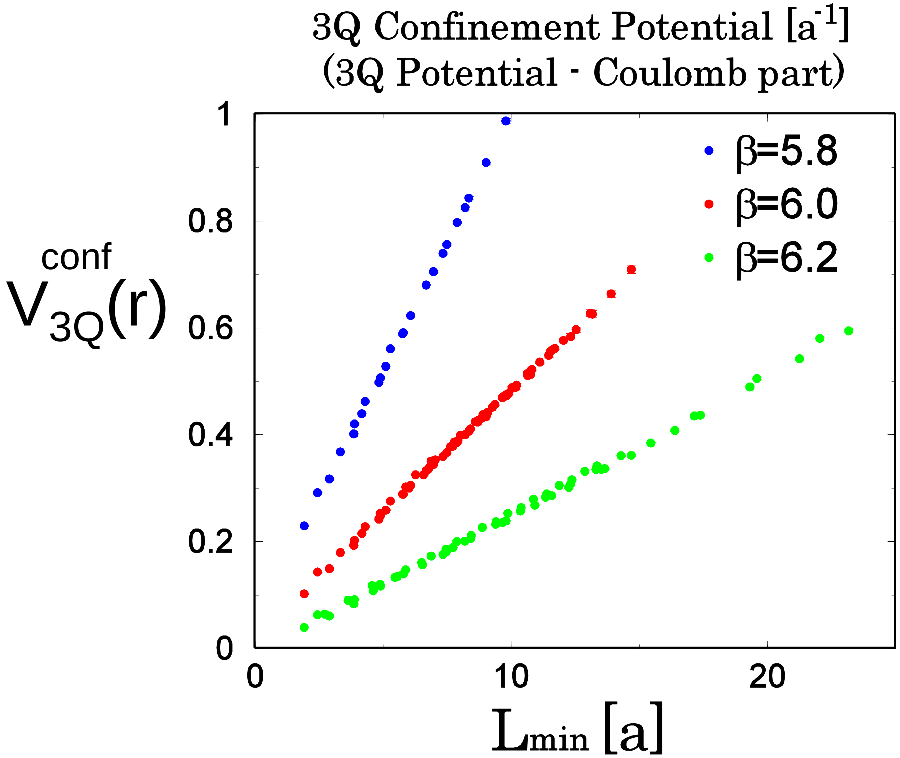
with $U_k \equiv P\exp\{ig\int_{\Gamma_k} dx_\mu A^\mu(x)\}$

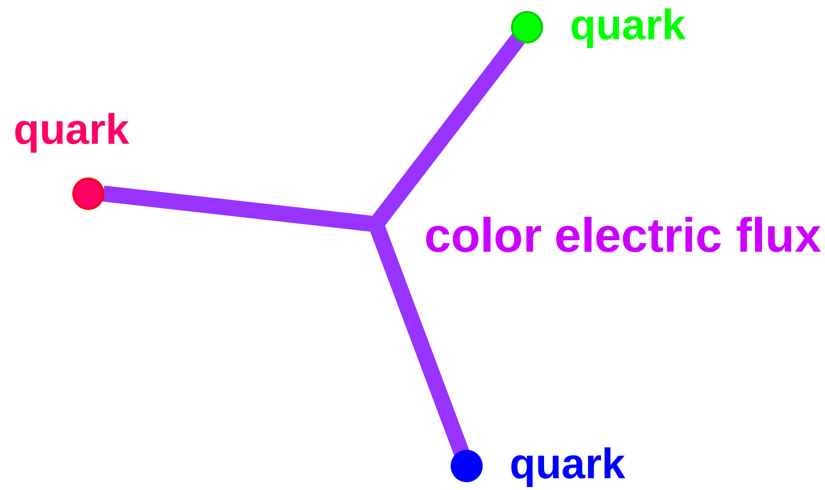


The 3Q Wilson loop physically means that a gauge-invariant 3Q state is created at $t = 0$ and is annihilated at $t = T$ with the three quarks spatially fixed in $\mathbf{R}^3$ for $0 < t < T$.

3 Quark Potential in Lattice QCD

PRL86 (2001) 18
PRD65 (2002) 114509
PRL 90 (2003)
PRD70 (2004) 074506
PRD72 (2005) 014505





***L_{min}* : total length of string linking three valence quarks**

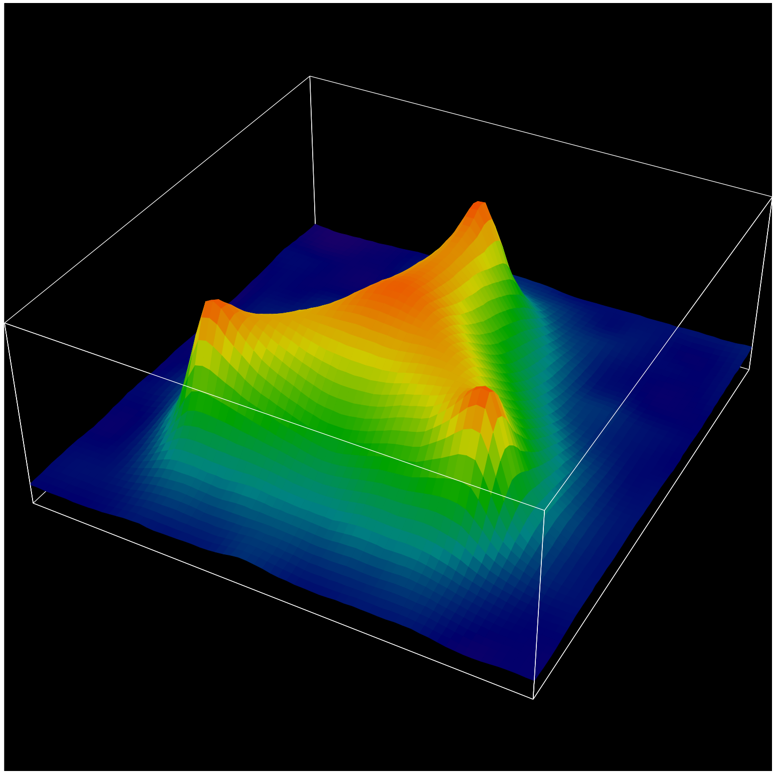
We analyze more than 300 different patterns of 3Q systems, and determine the functional form of *three-quark potential*.

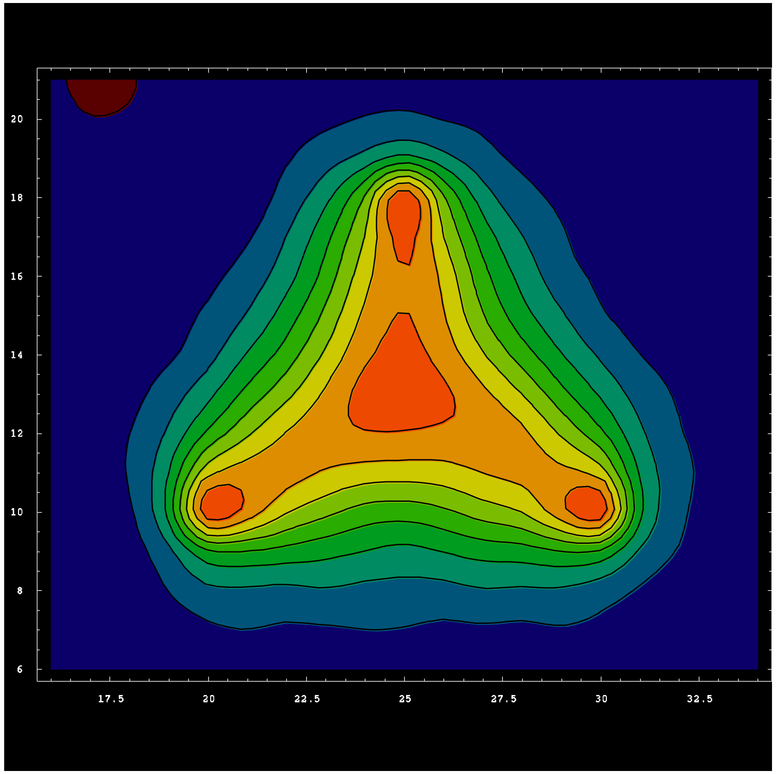
$$V_{3Q}(r) = \frac{g^2}{4\pi} \sum_{i<j}^{3} \frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma L_{min}$$

One-Gluon-Exchange Coulomb potential

Linear potential based on string picture

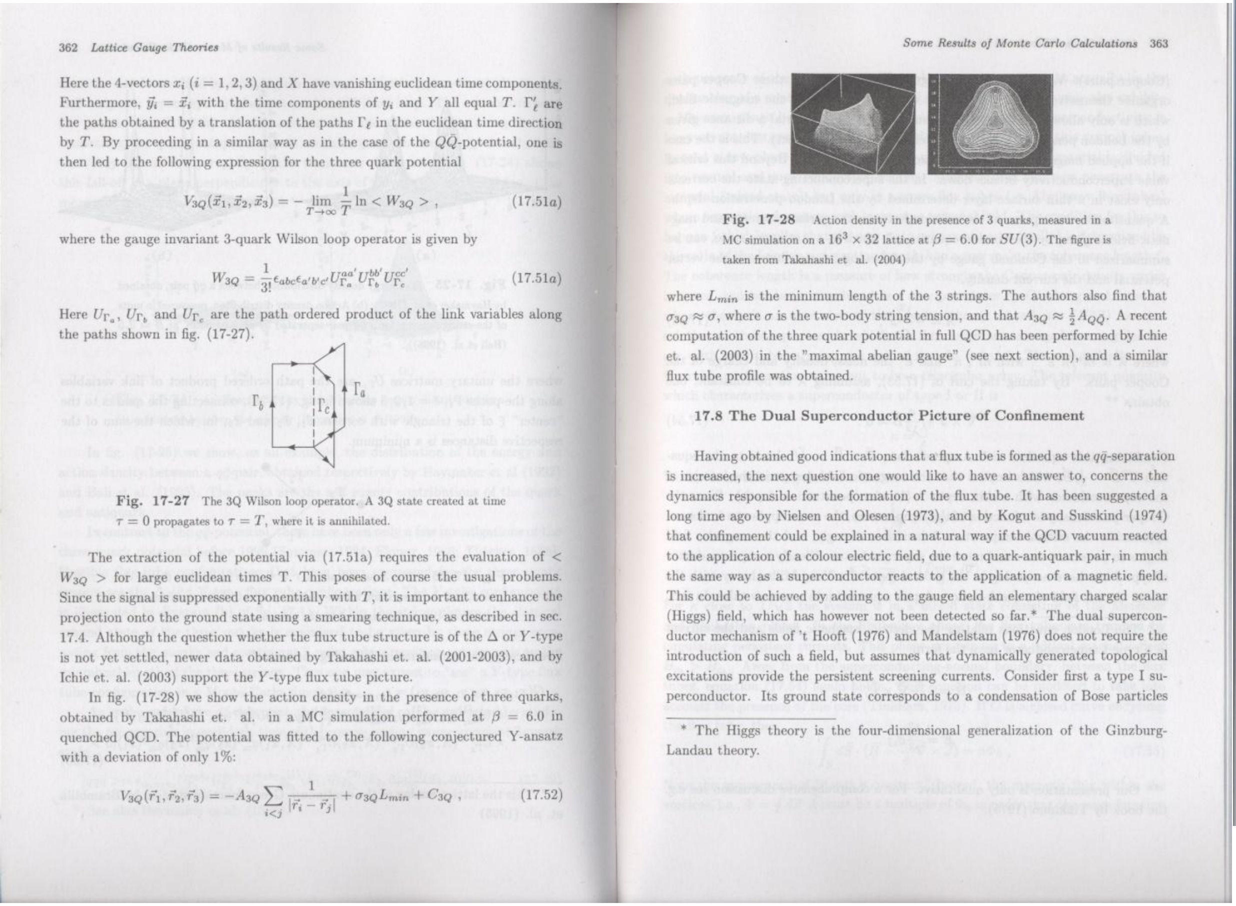
Lattice QCD result for Color Flux-Tube Formation in baryons





H. Ichie et al., Nucl. Phys. A721, 899 (2003)

The status of our studies of 3Q potential



Here the 4-vectors x_i ($i = 1, 2, 3$) and X have vanishing euclidean time components. Furthermore, $\vec{y}_i = \vec{x}_i$ with the time components of y_i and Y all equal T. Γ'_ℓ are the paths obtained by a translation of the paths Γ_ℓ in the euclidean time direction by T. By proceeding in a similar way as in the case of the $Q\bar{Q}$-potential, one is then led to the following expression for the three quark potential

$$V_{3Q}(\vec{x}_1, \vec{x}_2, \vec{x}_3) = -\lim_{T\to\infty}\frac{1}{T}\ln < W_{3Q} > , \qquad (17.51a)$$

where the gauge invariant 3-quark Wilson loop operator is given by

$$W_{3Q} = \frac{1}{3!}\epsilon_{abc}\epsilon_{a'b'c'}U^{aa'}_{\Gamma_a}U^{bb'}_{\Gamma_b}U^{cc'}_{\Gamma_c} \qquad (17.51a)$$

Here U_{Γ_a}, U_{Γ_b} and U_{Γ_c} are the path ordered product of the link variables along the paths shown in fig. (17-27).

Γ_b Γ_c Γ_a

Fig. 17-27 The 3Q Wilson loop operator. A 3Q state created at time $\tau = 0$ propagates to $\tau = T$, where it is annihilated.

The extraction of the potential via (17.51a) requires the evaluation of $< W_{3Q} >$ for large euclidean times T. This poses of course the usual problems. Since the signal is suppressed exponentially with T, it is important to enhance the projection onto the ground state using a smearing technique, as described in sec. 17.4. Although the question whether the flux tube structure is of the Δ or Y-type is not yet settled, newer data obtained by Takahashi et. al. (2001-2003), and by Ichie et. al. (2003) support the Y-type flux tube picture.

In fig. (17-28) we show the action density in the presence of three quarks, obtained by Takahashi et. al. in a MC simulation performed at $\beta = 6.0$ in quenched QCD. The potential was fitted to the following conjectured Y-ansatz with a deviation of only 1%:

$$V_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = -A_{3Q}\sum_{i<j}\frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma_{3Q}L_{min} + C_{3Q} , \qquad (17.52)$$

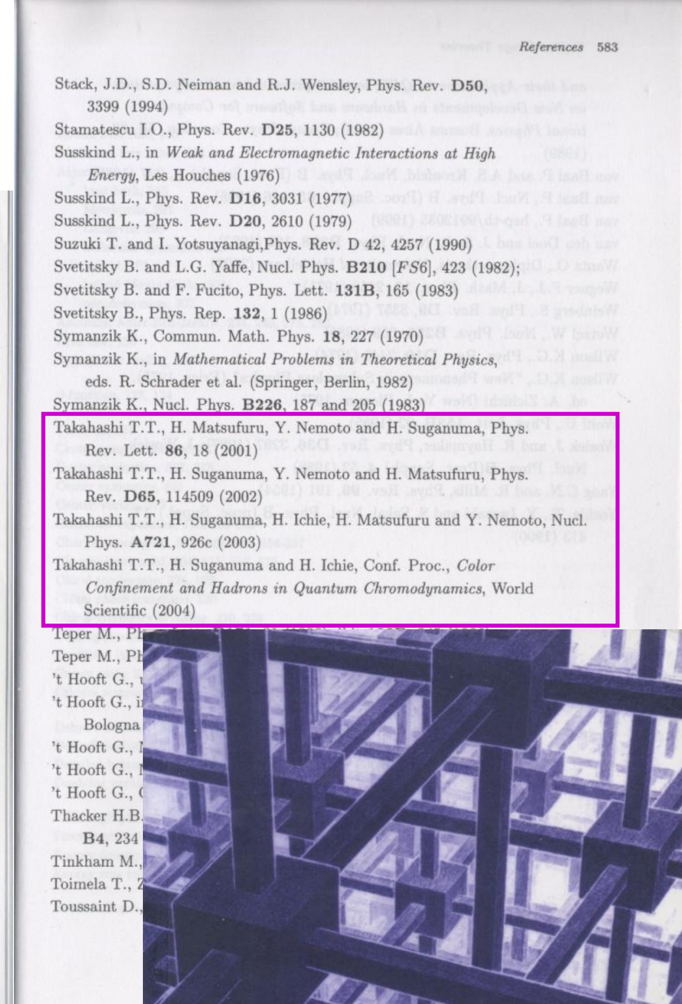
Fig. 17-28 Action density in the presence of 3 quarks, measured in a MC simulation on a $16^3 \times 32$ lattice at $\beta = 6.0$ for $SU(3)$. The figure is taken from Takahashi et al. (2004)

where L_{min} is the minimum length of the 3 strings. The authors also find that $\sigma_{3Q} \approx \sigma$, where σ is the two-body string tension, and that $A_{3Q} \approx \frac{1}{2}A_{Q\bar{Q}}$. A recent computation of the three quark potential in full QCD has been performed by Ichie et. al. (2003) in the "maximal abelian gauge" (see next section), and a similar flux tube profile was obtained.

17.8 The Dual Superconductor Picture of Confinement

Having obtained good indications that a flux tube is formed as the $q\bar{q}$-separation is increased, the next question one would like to have an answer to, concerns the dynamics responsible for the formation of the flux tube. It has been suggested a long time ago by Nielsen and Olesen (1973), and by Kogut and Susskind (1974) that confinement could be explained in a natural way if the QCD vacuum reacted to the application of a colour electric field, due to a quark-antiquark pair, in much the same way as a superconductor reacts to the application of a magnetic field. This could be achieved by adding to the gauge field an elementary charged scalar (Higgs) field, which has however not been detected so far.* The dual superconductor mechanism of 't Hooft (1976) and Mandelstam (1976) does not require the introduction of such a field, but assumes that dynamically generated topological excitations provide the persistent screening currents. Consider first a type I superconductor. Its ground state corresponds to a condensation of Bose particles

* The Higgs theory is the four-dimensional generalization of the Ginzburg-Landau theory.



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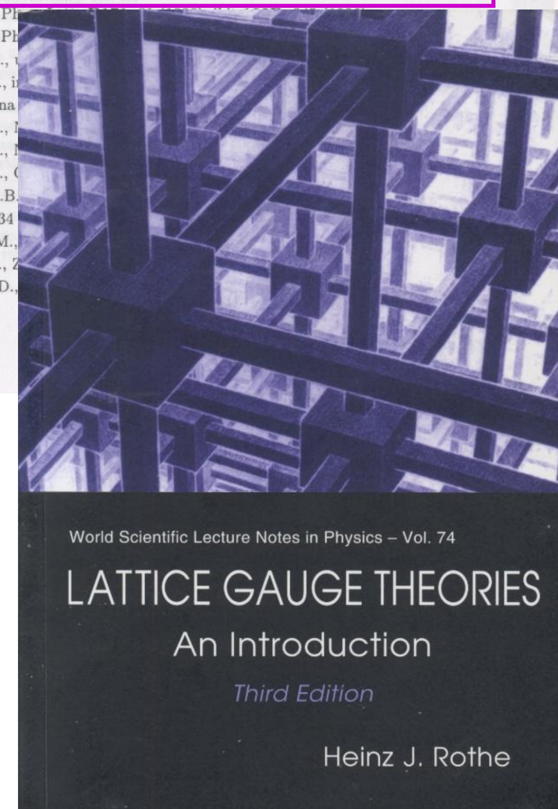
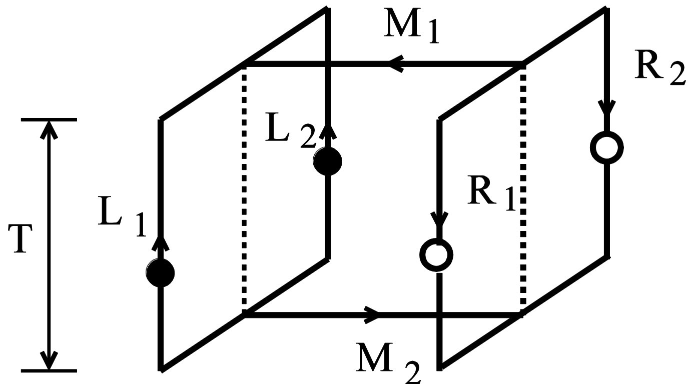


Our studies of the 3Q potential are introduced as "one whole subsection" with citing 4 our papers in 3rd edition of "Lattice Gauge Theories"(2005), which is one of the most popular lattice QCD text books.

First Lattice QCD Study for Static Quark Potential in Multi-Quark System

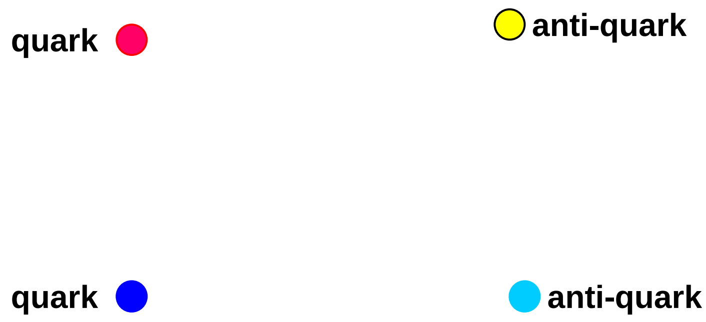
We formulate Multi-Quark Wilson Loops.

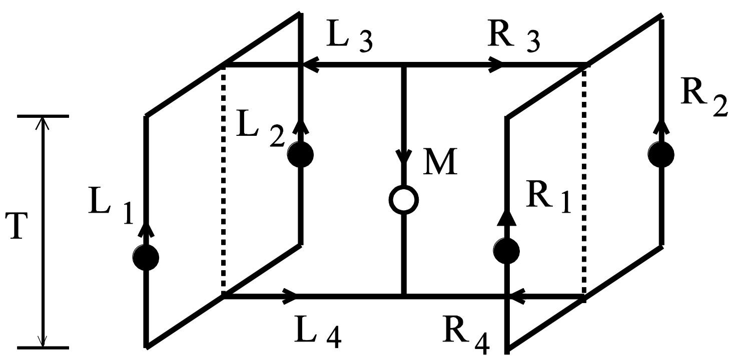
Okiharu, H.S. et al. PRL 94 (2005) 192001
Okiharu, H.S. et al. PRD72 (2005) 014505



4 Quark Wilson Loop

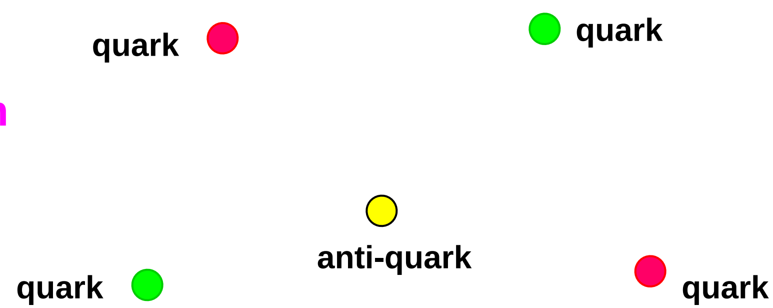
4 quark system





5 Quark Wilson Loop

5 quark system



$$V_{NQ}(r) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W_{NQ} \rangle_T$$

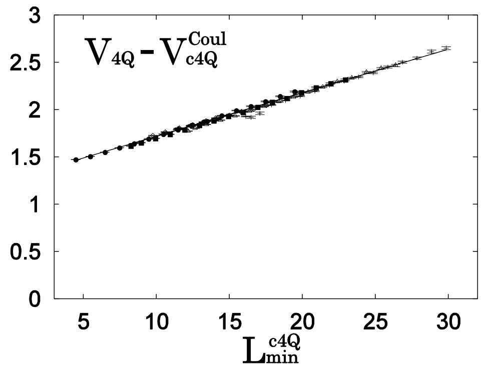
The *Multi-Quark potentials* can be obtained from the corresponding *Multi-Quark Wilson Loops*.

First Lattice QCD Study for Static Quark Potential in Multi-Quark System

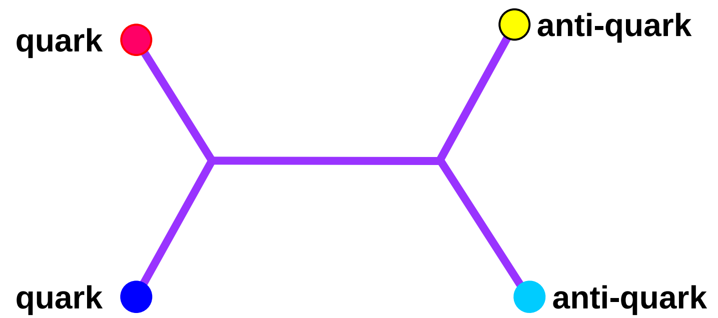
Okiharu, H.S. et al. PRL 94 (2005) 192001
Okiharu, H.S. et al. PRD72 (2005) 014505

$V_{NQ}(r)$

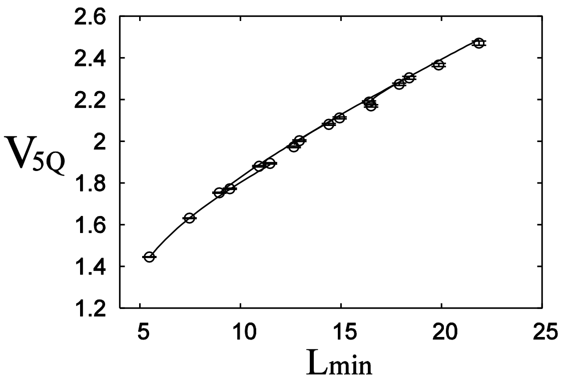
N=4,5



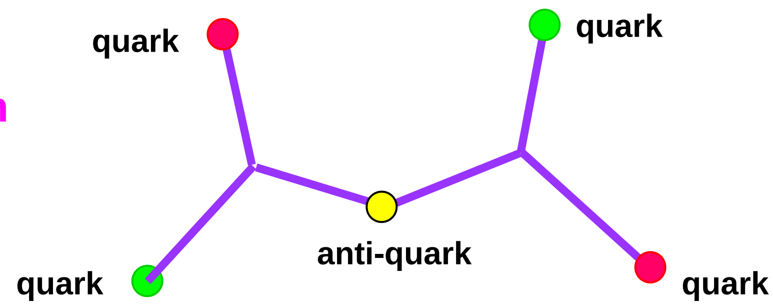
4 quark system



color flux tube



5 quark system



Partial lattice QCD data of Multi-quark potential

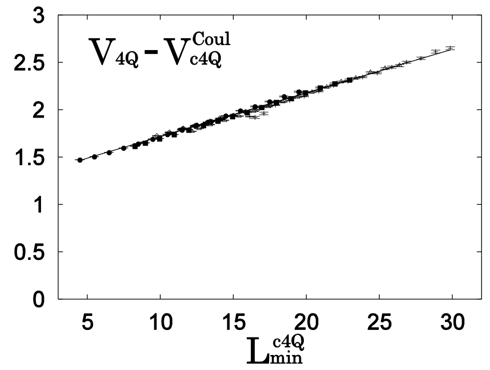
For more than 200 different patterns of multi-quark configurations, we have accurately performed the first lattice QCD calculations for *multi-quark potentials*.

First Lattice QCD Study for Static Quark Potential in Multi-Quark System

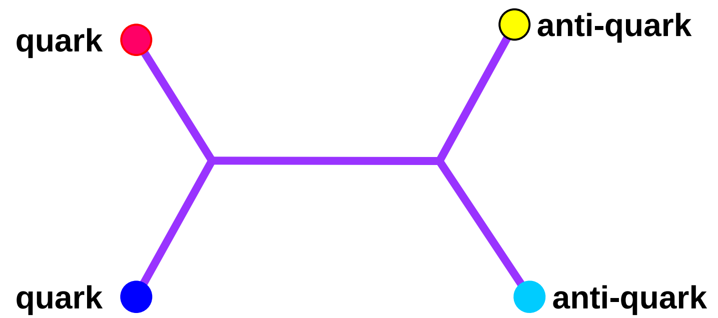
Okiharu, H.S. et al. PRL 94 (2005) 192001
Okiharu, H.S. et al. PRD72 (2005) 014505

$V_{NQ}(r)$

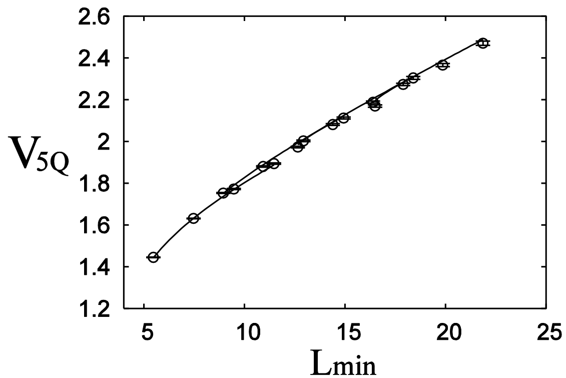
N=4,5



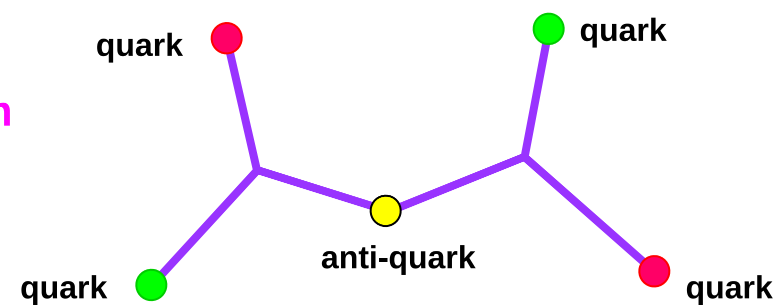
4 quark system



color flux tube



5 quark system



L_{min}* : *total length of string linking the N valence quarks

$$V_{NQ}(r) = \frac{g^2}{4\pi}\sum_{i<j}^{N}\frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma L_{min}$$

One-Gluon-Exchange Coulomb potential

Linear potential based on string picture

Flip-Flop in Tetra-Quark System

We quantitatively study the tetra-quark (4Q) potential for the QQ-$\bar{Q}\bar{Q}$ systems in SU(3) lattice QCD, and find the "flip-flop", i.e., the recombination of the flux-tube, between a connected 4Q state and a "two-meson" state around the level-crossing point.

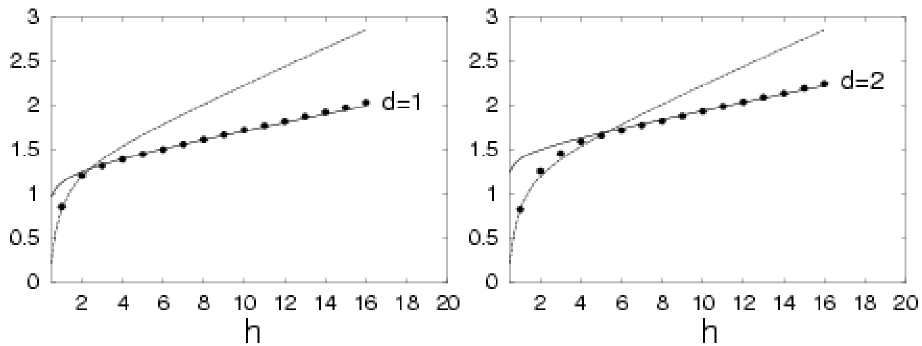


Figure 1: The typical lattice QCD results for the flip-flop between the connected 4Q state and the two-meson state for the planar configuration. The symbols denote lattice QCD results. The curves describe the theoretical form: the solid curves denote the OGE plus multi-Y Ansatz for connected 4Q states, and the dashed curves the two-meson Ansatz.

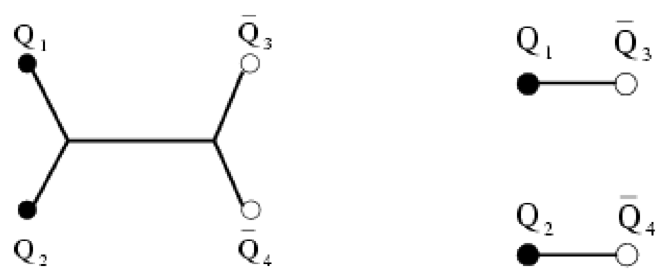


Figure 2: (a) The connected tetraquark system. All quarks and antiquarks are connected with the single flux-tube, which is double-Y-shaped. (b) The disconnected tetraquark system, which corresponds to a "two-meson" state.

Summary of Static Potentials

We have performed the first accurate Lattice QCD studies for *static multi-quark (3Q, 4Q, 5Q) potentials*.

The multi-quark potential is well described by
OGE Coulomb+ String-picture Linear Confinement Potential.

$$V_{NQ}(r) = \frac{g^2}{4\pi} \sum_{i<j}^{N} \frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma L_{min}$$

One-Gluon-Exchange Coulomb potential

Linear potential based on string picture

L_{min} : *total length of string linking the N valence quarks*

We have found the Universality of Quark Confinement Force (String Tension) in hadrons: $\sigma_{Q\bar{Q}} = \sigma_{3Q} = \sigma_{4Q} = \sigma_{5Q}$

Multi-Quark Hamiltonian based on QCD

In this way, from the Lattice QCD studies for the multi-quark potential, we obtan Multi-Quak Hamiltonian based on QCD:

$$H_{NQ} = \sum_{i=1}^{N} (p_i^2 + M_i^2)^{1/2} + \frac{g^2}{4\pi} \sum_{i<j}^{N} \frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma L_{min}$$

One-Gluon-Exchange Coulomb potential

Linear potential based on string picture

OGE Coulomb+ String-picture Linear Confinement Potential

L_{min} : *total length of string linking the N valence quarks*

Lattice-QCD based Quark model calculation
is possible for Multi-Quark system.

e.g. “Four- and Five-Body Scattering Calculations of Exotic Hadron Systems”
E.Hiyama, H.S., M.Kamimura, Prog. Theor. Phys. Suppl. 168 (2007) 101-106.

Strategy to understand hadron properties from QCD

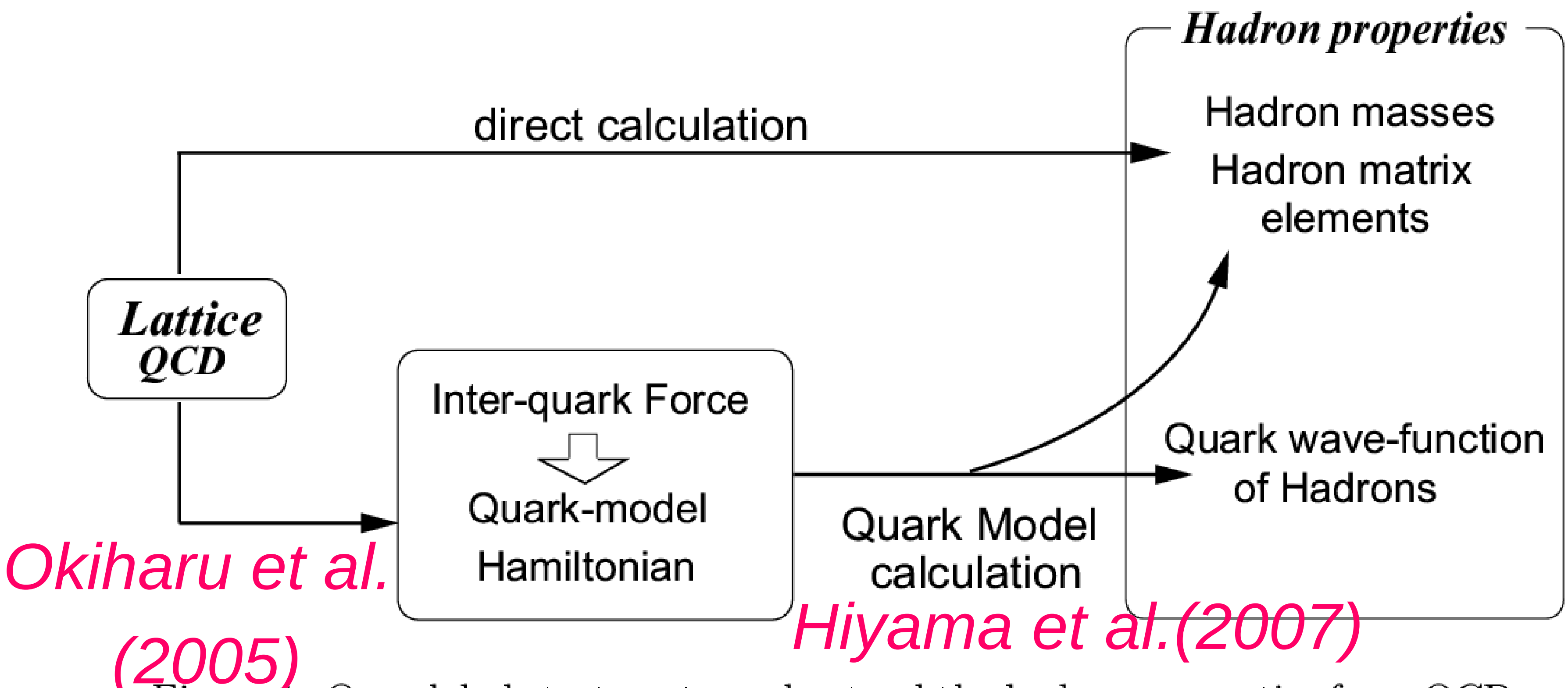


Figure 1: Our global strategy to understand the hadron properties from QCD. One way is the direct lattice QCD calculations for the low-lying hadron masses and simple hadron matrix elements, although the wave function is unknown and the practically calculable quantities are severely limited. The other way is to construct the quark model from QCD. From the analysis of the inter-quark forces in lattice QCD, we extract the quark-model Hamiltonian. Through the quark model calculation, one can obtain the quark wave-function of hadrons and more complicated properties of hadrons including properties of excited hadrons.

Cheap summary and trivial outlook

- Lattice QCD studies for multi-quark physics is rather difficult and still on-going, and still one of the challenging topics.
- In these years, facing the experimental reports on multi-quark hadrons, lattice QCD physicists also faced new-type calculations and gradually improved their skills.
- But, to tell the truth, *only several lattice groups* have *partially* committed this new subject of multi-quark physics.
- If *Facing serious experimental discoveries of multi-quarks*, many lattice groups must start this new interesting subject, and drastic improvements and solid lattice-QCD predictions must be expected for the new exotic hadrons.

Most lattice people are waiting for more experimental discoveries!